EOB/NR: WHERE ARE WE GOING?

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EFFECTIVE-ONE-BODY (EOB)

approach to the general relativistic two-body problem

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

key ideas:

(1) Replace two-body dynamics (m_1, m_2) by dynamics of a particle $(\mu \equiv m_1 m_2/(m_1 + m_2))$ in an effective metric $g^{\rm eff}_{\mu\nu}(u)$, with

$$u \equiv GM/c^2 R\,, \quad M \equiv m_1 + m_2$$

(2) Systematically use RESUMMATION of PN expressions (both $g^{\rm eff}_{\mu\nu}$ and $\mathcal{F}_{RR}$) based on various physical requirements

(3) Require continuous deformation w.r.t. $\nu \equiv \mu/M \equiv m_1 m_2/(m_1 + m_2)^2$ in the interval $0 \leq \nu \leq \frac{1}{4}$

STRUCTURE OF THE EOB FORMALISM

PN dynamics
(DD81,D82,DJS01,IF03,BDIF04)

PN rad losses
WW76, BDIWW95, BDEFI 05

PN waveform
BD89, B95&05,ABIQ04,

BH perturbations
RW57, Z70, Z72

Resummed (BD99)

Resummed (DIS98)

Resummed (DN07,DIN08)

QNMs spectrum
$\sigma_N = \alpha_N + i\omega_N$

EOB Hamiltonian
$H_{\rm EOB}$

EOB Rad. Reac. force
$\hat{\mathcal{F}}_\varphi$

Factorized waveform

Matching at merger time

$$h_{\ell m} = h^{(N,\epsilon)}_{\ell m}\hat{h}^{(\epsilon)}_{\ell m}$$
$$\hat{h}^{(\epsilon)}_{\ell m} = \hat{S}^{(\epsilon)}_{eff} T_{\ell m} e^{i\delta_{\ell m}} \rho^{\ell}_{\ell m}$$

$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}},$$
$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r},$$
$$\Omega \equiv \frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi},$$
$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi .$$

BNS: tides
(Love numbers)

$$h^{\rm ringdown}_{\ell m}(t) = \sum_N C^+_N e^{-\sigma^+_N (t-t_m)}$$

EOB waveform

$$h^{\rm EOB}_{\ell m} = \theta(t_m - t) h^{\rm insplunge}_{\ell m}(t) + \theta(t - t_m) h^{\rm ringdown}_{\ell m}(t)$$

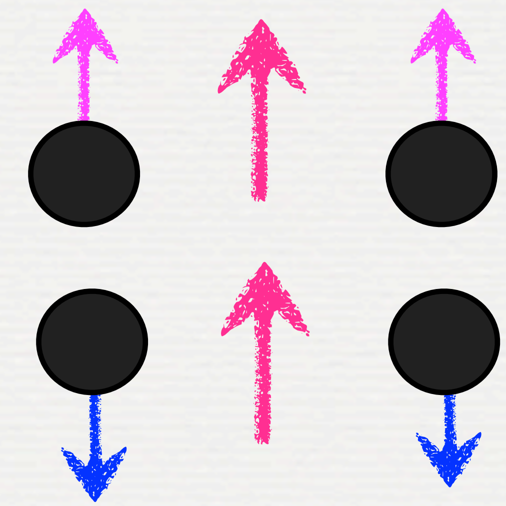
SPINNING BBHS

Spin-orbit & spin-spin couplings

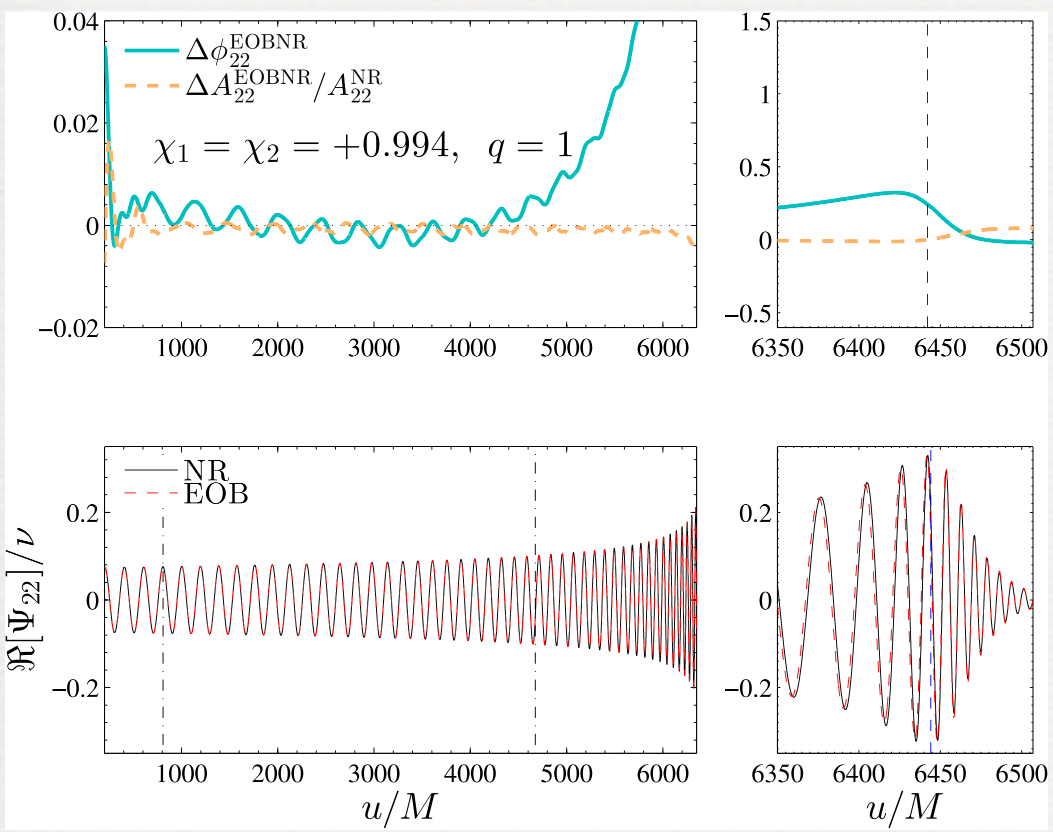
(i) Spins aligned with L: repulsive (slower) L-o-n-g-e-r INSPIRAL

(ii) Spins anti-aligned with L: attractive (faster) Shorter INSPIRAL

(iii) Misaligned spins: precession of the orbital plane (waveform modulation)



$$\chi_{1,2} = \frac{c\,\mathbf{S}_{1,2}}{Gm_{1,2}^2}$$

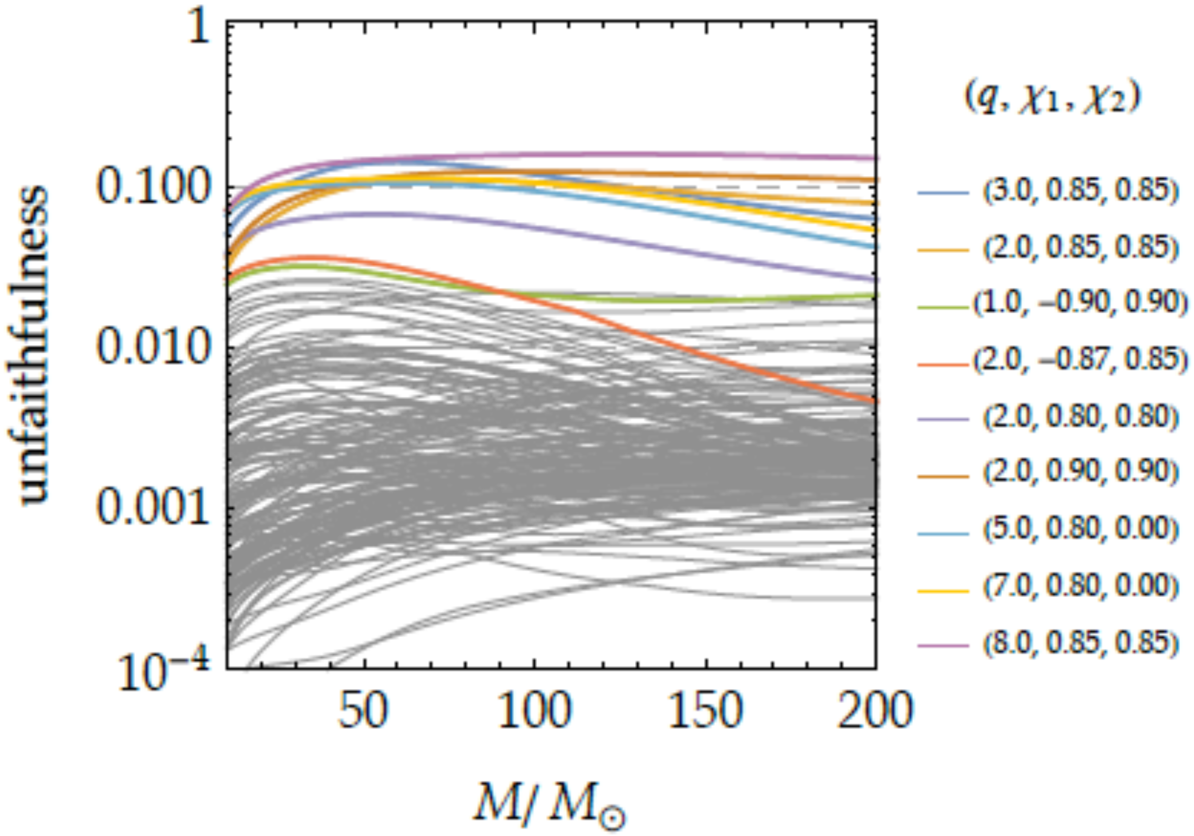


EOB/NR agreement: sophisticated (though rather simple) model for spin-aligned binaries

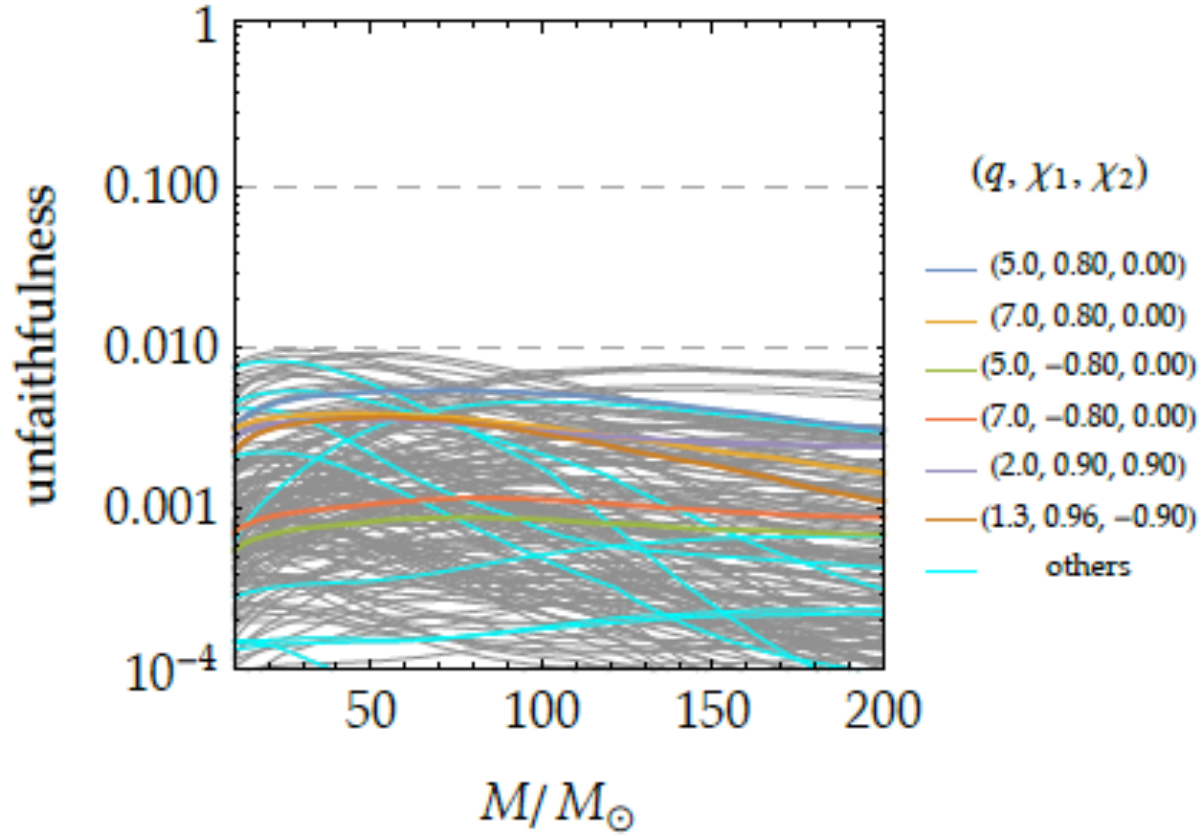
Damour&Nagar, PRD90 (2014), 024054 (Hamiltonian)
Damour&Nagar, PRD90 (2014), 044018 (Ringdown)
Nagar,Damour, Reisswig & Pollney, PRD 93 (2016), 044046

AEI model, SEOBNRv4, Bohe et al., arXiv:1611.03703v1 (PRD in press)

SEOBNRv2



SEOBNRv4



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$$d_{SO} = +147.481449\chi^3\nu^2 - 568.651115\chi^3\nu + 66.198703\chi^3 - 343.313058\chi^2\nu + 2495.293427\chi\nu^2 - 44.532373\,,$$

$$d_{SS} = +528.511252\chi^3\nu^2 - 41.000256\chi^3\nu + 1161.780126\chi^2\nu^3 - 326.324859\chi^2\nu^2 + 37.196389\chi\nu + 706.958312\nu^3 - 36.027203\nu + 6.068071\,,$$

AEI model: arXiv: 1611.03703v1

Strong recalibration of the state-of-the-art SEOBNRv2 model (used for O1) to be faithful towards a set of 141 NR simulations (about 100 new ones)

More NR simulations are essential to "calibrate & improve" the model

SO & SS EFFECTS IN EOB HAMILTONIAN

New way of combining available knowledge within some Hamiltonian
[Damour&Nagar, PRD 2014]

$$\hat{H}_{\rm eff} = \frac{g_S^{\rm eff}}{r^3}\mathbf{L}\cdot\mathbf{S} + \frac{g_{S^*}^{\rm eff}}{r^3}\mathbf{L}\cdot\mathbf{S}^* + \sqrt{A(1+\gamma^{ij}p_ip_j + Q_4(p))}$$

with the structure

$$g_S^{\rm eff} = 2 + \nu(\text{PN corrections}) + (\text{spin})^2\text{corrections}$$

$$g_{S^*}^{\rm eff} = \left(\frac{3}{2} + \text{test mass coupling}\right) + \nu(\text{PN corrections}) + (\text{spin})^2\text{corrections}$$

$$A = 1 - \frac{2}{r} + \nu(\text{PN corrections}) + (\text{spin})^2\text{corrections}$$

$$\gamma^{ij} = \gamma^{ij}_{\rm Kerr} + \nu(\text{PN corrections}) + \dots$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = M^2(X_1^2\chi_1 + X_2^2\chi_2)$$

$$X_i = m_i/M$$

$$\mathbf{S}^* = \frac{m_2}{m1}\mathbf{S}_1 + \frac{m_1}{m_2}\mathbf{S}_2 = M^2\nu(\chi_1+\chi_2)$$

$$-1 \leq \chi_i \leq 1$$

THE TWO TYPES OF SPIN-ORBIT COUPLINGS

$$\hat{H}^{\rm eff}_{\rm SO} = G_S \mathbf{L}\cdot\mathbf{S} + G_{S*}\mathbf{L}\cdot\mathbf{S}^* \qquad G_S = \frac{1}{r^3}g_S^{\rm eff}, \quad G_{S*} = \frac{1}{r^3}g_{S*}^{\rm eff}$$

In the Kerr limit, only S-type gyro-gravitomagnetic ratio enters:

$$g_S^{\rm eff} = 2\frac{r^2}{r^2+a^2\left[(1-\cos^2\theta)\left(1+\frac{2}{r}\right)+2\cos^2\theta\right]+\frac{a^4}{r^2}\cos^2\theta} = 2+\mathcal{O}[({\rm spin})^2]$$

PN calculations yield (in some spin gauge)[DJS08, Hartung&Steinhoff11,Nagar11,Barausse&Buonanno11]

"Effective" NNNLO SO-coupling

$$g_S^{\rm eff} = 2+\frac{1}{c^2}\left\{-\frac{1}{r}\frac{5}{8}\nu-\frac{33}{8}(\mathbf{n}\cdot\mathbf{p})^2\right\}$$
$$+\frac{1}{c^4}\left\{-\frac{1}{r^2}\left(\frac{51}{4}\nu+\frac{\nu^2}{8}\right)+\frac{1}{r}\left(-\frac{21}{2}\nu+\frac{23}{8}\nu^2\right)(\mathbf{n}\cdot\mathbf{p})^2+\frac{5}{8}\nu(1+7\nu)(\mathbf{n}\cdot\mathbf{p})^4\right\}, \quad +\frac{1}{c^6}\frac{\nu c_3}{r^3}$$

$$g_{S^*}^{\rm eff} = \frac{3}{2}+\frac{1}{c^2}\left\{-\frac{1}{r}\left(\frac{9}{8}+\frac{3}{4}\nu\right)-\left(\frac{9}{4}\nu+\frac{15}{8}\right)(\mathbf{n}\cdot\mathbf{p})^2\right\}$$
$$+\frac{1}{c^4}\left\{-\frac{1}{r^2}\left(\frac{27}{16}+\frac{39}{4}\nu+\frac{3}{16}\nu^2\right)+\frac{1}{r}\left(\frac{69}{16}-\frac{9}{4}\nu+\frac{57}{16}\nu^2\right)(\mathbf{n}\cdot\mathbf{p})^2+\left(\frac{35}{16}+\frac{5}{2}\nu+\frac{45}{16}\nu^2\right)(\mathbf{n}\cdot\mathbf{p})^4\right\} +\frac{1}{c^6}\frac{\nu c_3}{r^3}$$

The NR-informed effective parameter makes the spin-orbit coupling stronger or weaker with respect to the simple analytical prediction

40 NR SXS Datasets (public in the fall of 2013 and used before for SEOBNRv2)

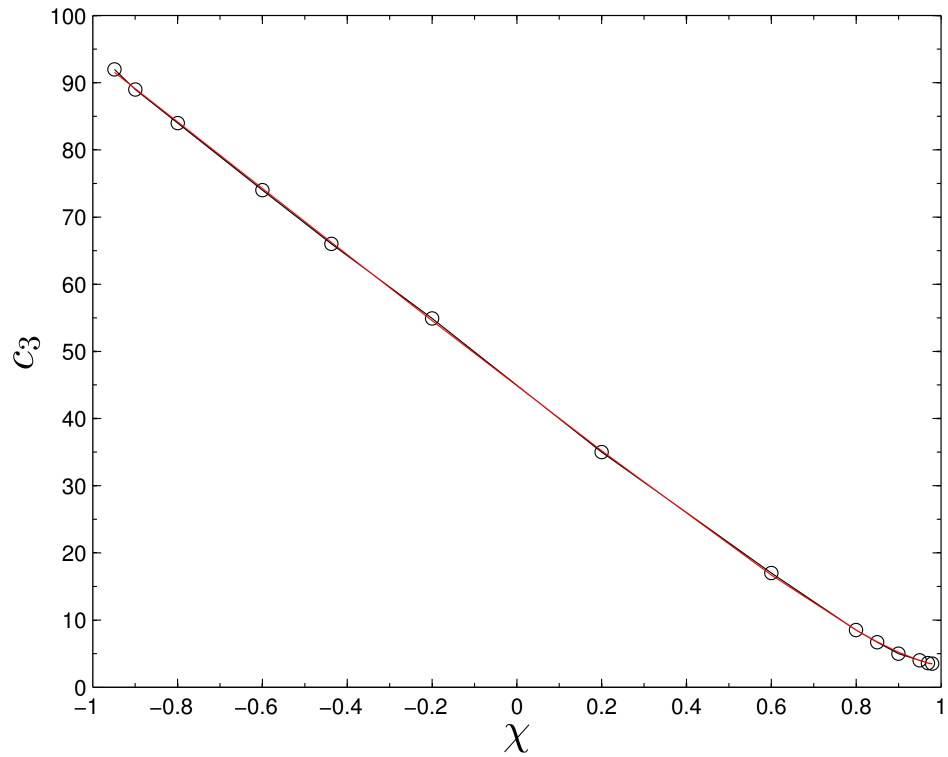
TABLE I: EOB/NR phasing comparison. The columns report: the number of the dataset; the name of the configuration in the SXS catalog; the mass ratio $q = m_1/m_2$; the symmetric mass ratio ν; the dimensionless spins χ_1 and χ_2; the phase difference $\Delta\phi^{\rm EOBNR} \equiv \phi^{\rm EOB} - \phi^{\rm NR}$ computed at NR merger; the NR phase uncertainty at NR merger $\delta\phi^{\rm NR}_{\rm mrg}$ (when available) measured taking the difference between the two highest resolution levels (see text); the maximum value of the unfaithfulness $\bar{F} \equiv 1 - F$ as per Eq. (22). The $\Delta\phi^{\rm EOBNR}$'s in brackets for $\chi_1 = \chi_2 > +0.85$ were obtained using Eq. (21) for $\Delta t^{\rm NQC}(\chi)$.

#	Name	N orbits	q	ν	χ_1	χ_2	$\Delta\phi^{\rm EOBNR}_{\rm mrg}$ [rad]	$\delta\phi^{\rm NR}_{\rm mrg}$ [rad]	max($\bar{F}$)
1	SXS:BBH:none	14	1	0.25	0.0	0.0	−0.016	...	0.00087
2	SXS:BBH:0066	28	1	0.25	0.0	0.0	+0.010	...	0.00068
3	SXS:BBH:0002	32.42	1	0.25	0.0	0.0	+0.073	0.066	0.00101
4	SXS:BBH:0007	29.09	1.5	0.24	0	0	+0.05	0.018	0.00201
5	SXS:BBH:0169	15.68	2	$0.\bar{2}$	0	0	−0.15	0.02	0.00045
6	SXS:BBH:0030	18.22	3	0.1875	0	0	−0.074	0.087	0.00035
7	SXS:BBH:0167	15.59	4	0.16	0	0	−0.059	0.52	0.00035
8	SXS:BBH:0056	28.81	5	$0.13\bar{8}$	0	0	−0.089	0.44	0.00038
9	SXS:BBH:0166	21.56	6	0.1224	0	0	−0.198	...	0.00037
10	SXS:BBH:0063	25.83	8	0.0987	0	0	−0.453	1.01	0.00292
11	SXS:BBH:0185	24.91	9.98911	0.0827	0	0	−0.0051	0.376	0.00066
12	SXS:BBH:0004	30.19	1	0.25	−0.50	0.0	−0.017	0.068	0.00403
13	SXS:BBH:0005	30.19	1	0.25	+0.50	0.0	+0.08	0.28	0.00052
14	SXS:BBH:0156	12.42	1	0.25	−0.95	−0.95	+0.32	2.17	0.00058
15	SXS:BBH:0159	12.67	1	0.25	−0.90	−0.90	+0.06	0.38	0.00047
16	SXS:BBH:0154	13.24	1	0.25	−0.80	−0.80	+0.11	...	0.00044
17	SXS:BBH:0151	14.48	1	0.25	−0.60	−0.60	−0.049	0.14	0.00042
18	SXS:BBH:0148	15.49	1	0.25	−0.44	−0.44	+0.14	0.72	0.00043
19	SXS:BBH:0149	17.12	1	0.25	−0.20	−0.20	+0.45	0.90	0.00085
20	SXS:BBH:0150	19.82	1	0.25	+0.20	+0.20	+0.94	0.99	0.00275
21	SXS:BBH:0152	22.64	1	0.25	+0.60	+0.60	+0.01	0.36	0.00068
22	SXS:BBH:0155	24.09	1	0.25	+0.80	+0.80	−0.39	0.26	0.00110
23	SXS:BBH:0153	24.49	1	0.25	+0.85	+0.85	+0.06	...	0.00059
24	SXS:BBH:0160	24.83	1	0.25	+0.90	+0.90	+0.41 (+0.41)	0.80	0.00117
25	SXS:BBH:0157	25.15	1	0.25	+0.95	+0.95	+0.37 (+0.83)	1.18	0.00295
26	SXS:BBH:0158	25.27	1	0.25	+0.97	+0.97	+0.37 (+0.49)	1.26	0.00325
27	SXS:BBH:0172	25.35	1	0.25	+0.98	+0.98	+0.99 (+0.46)	2.02	0.00422
28	SXS:BBH:0177	25.40	1	0.25	+0.99	+0.99	+0.22 (+0.48)	0.40	0.00507
29	SXS:BBH:0178	25.43	1	0.25	+0.994	+0.994	+0.24 (+0.23)	−0.53	0.00506
30	SXS:BBH:0013	23.75	1.5	0.24	+0.5	0	+0.31	...	0.00058
31	SXS:BBH:0014	22.63	1.5	0.24	−0.5	0	−0.15	0.15	0.00046
32	SXS:BBH:0162	18.61	2	$0.\bar{2}$	+0.6	0	−0.20	0.71	0.00027
33	SXS:BBH:0036	31.72	3	0.1875	−0.5	0	+0.08	0.065	0.00040
34	SXS:BBH:0031	21.89	3	0.1875	+0.5	0	+0.12	0.034	0.00023
35	SXS:BBH:0047	22.72	3	0.1875	+0.5	+0.5	−0.034	...	0.00030
36	SXS:BBH:0046	14.39	3	0.1875	−0.5	−0.5	+0.36	...	0.00054
37	SXS:BBH:0110	24.24	5	$0.13\bar{8}$	+0.5	0	+0.24	...	0.00016
38	SXS:BBH:0060	23.17	5	$0.13\bar{8}$	−0.5	0	+0.21	0.8	0.00034
39	SXS:BBH:0064	19.16	8	0.0987	−0.5	0	+0.026	0.8	0.00042
40	SXS:BBH:0065	33.97	8	0.0987	+0.5	0	+1.33	−3.0	0.00040

Several equal-mass, equal-spin data

Just a few unequal-mass, unequal-spin data

SPIN-ORBIT NR CALIBRATION

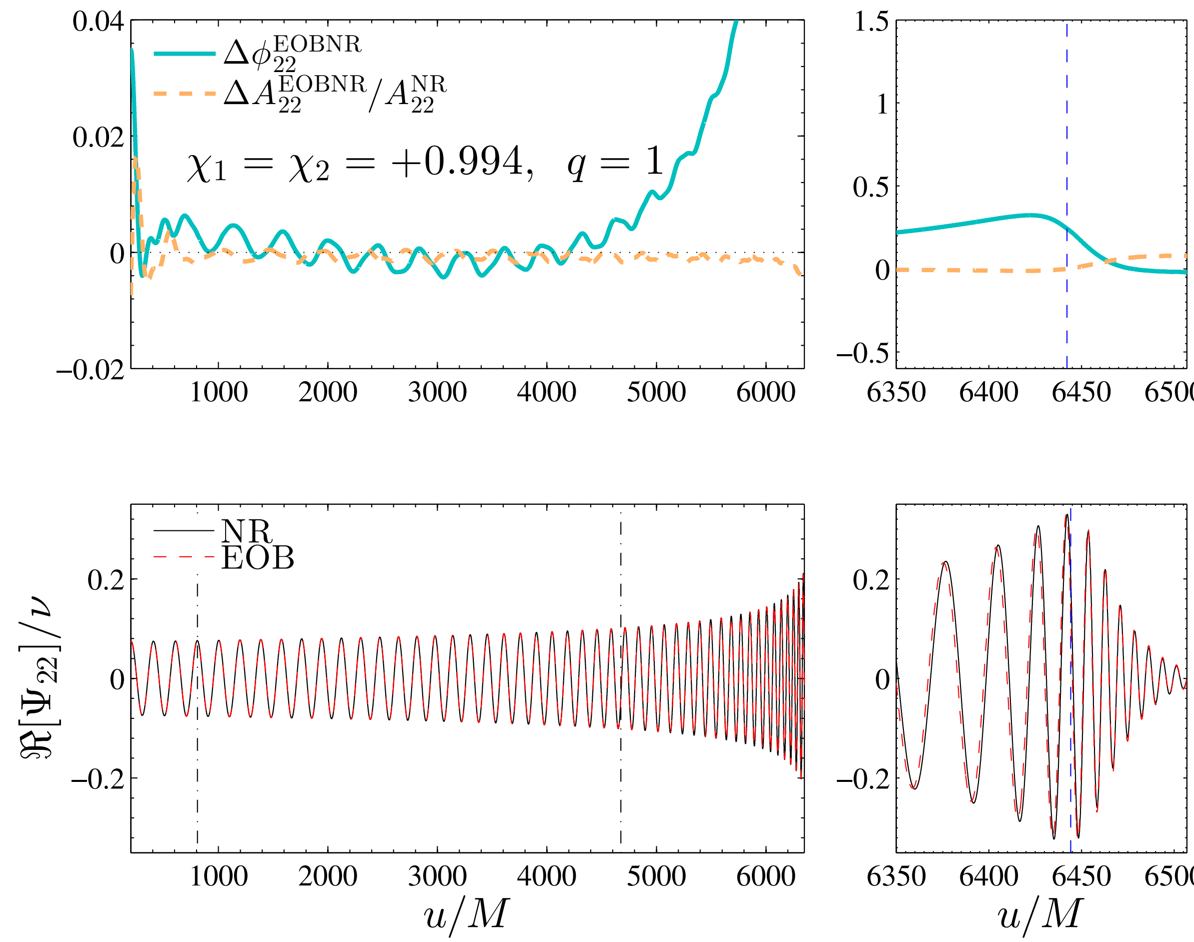


Quasi-linear function of the spins

$$c_3(\tilde{a}_1,\tilde{a}_2,\nu) = p_0\frac{1+n_1(\tilde{a}_1+\tilde{a}_2)+n_2(\tilde{a}_1+\tilde{a}_2)^2}{1+d_1(\tilde{a}_1+\tilde{a}_2)}$$
$$+(p_1\nu+p_2\nu^2+p_2\nu^3)(\tilde{a}_1+\tilde{a}_2)\sqrt{1-4\nu}$$
$$+p_4(\tilde{a}_1-\tilde{a}_2)\nu^2,$$

+ interpolating fits for NQC functioning point, ringdown coefficients etc. (Achille's heel...still ok..)

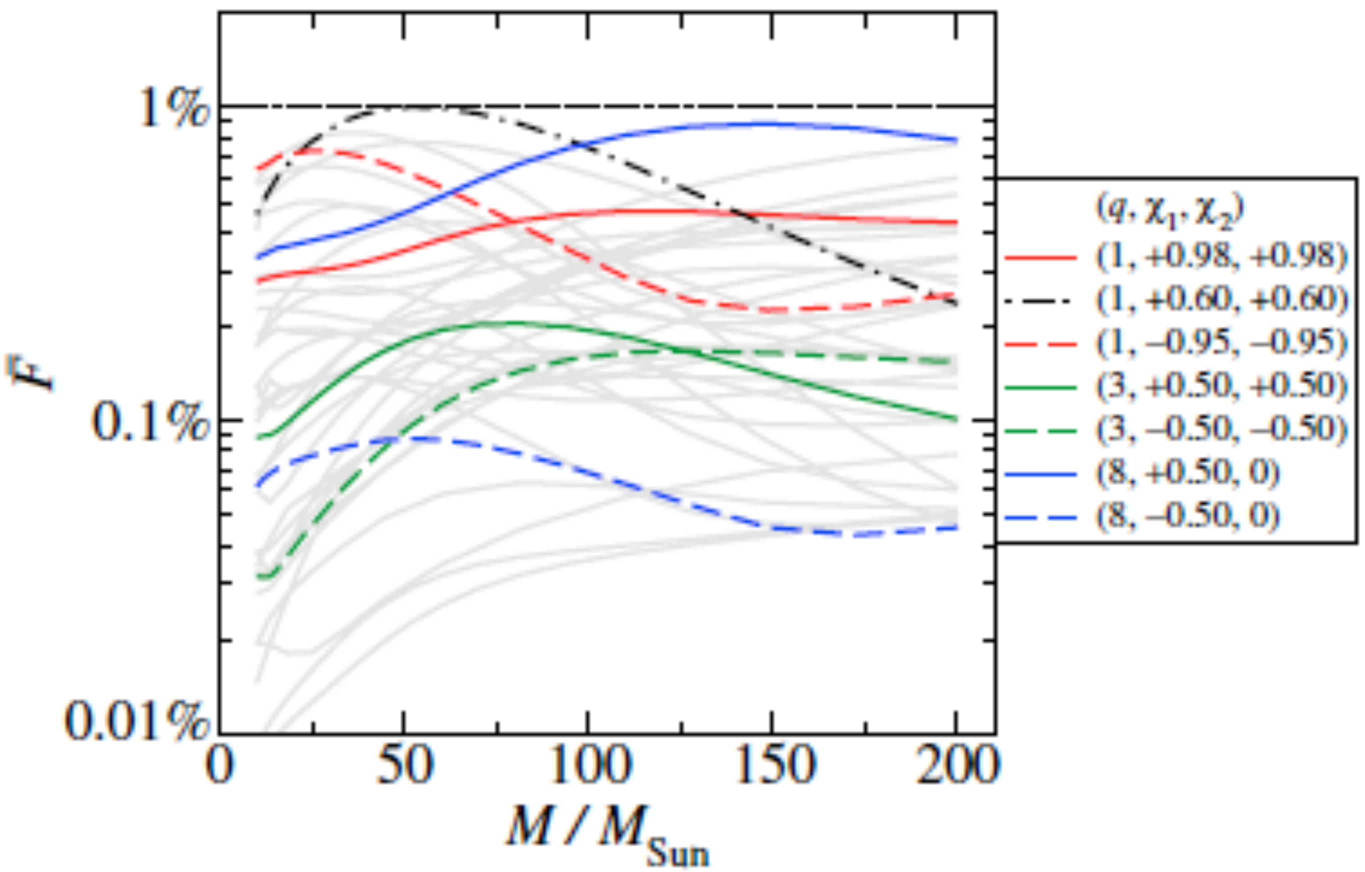
$$\tilde{a}_{1,2} = X_{1,2}\chi_{1,2}$$
$$X_{1,2} \equiv \frac{m_{1,2}}{M}$$



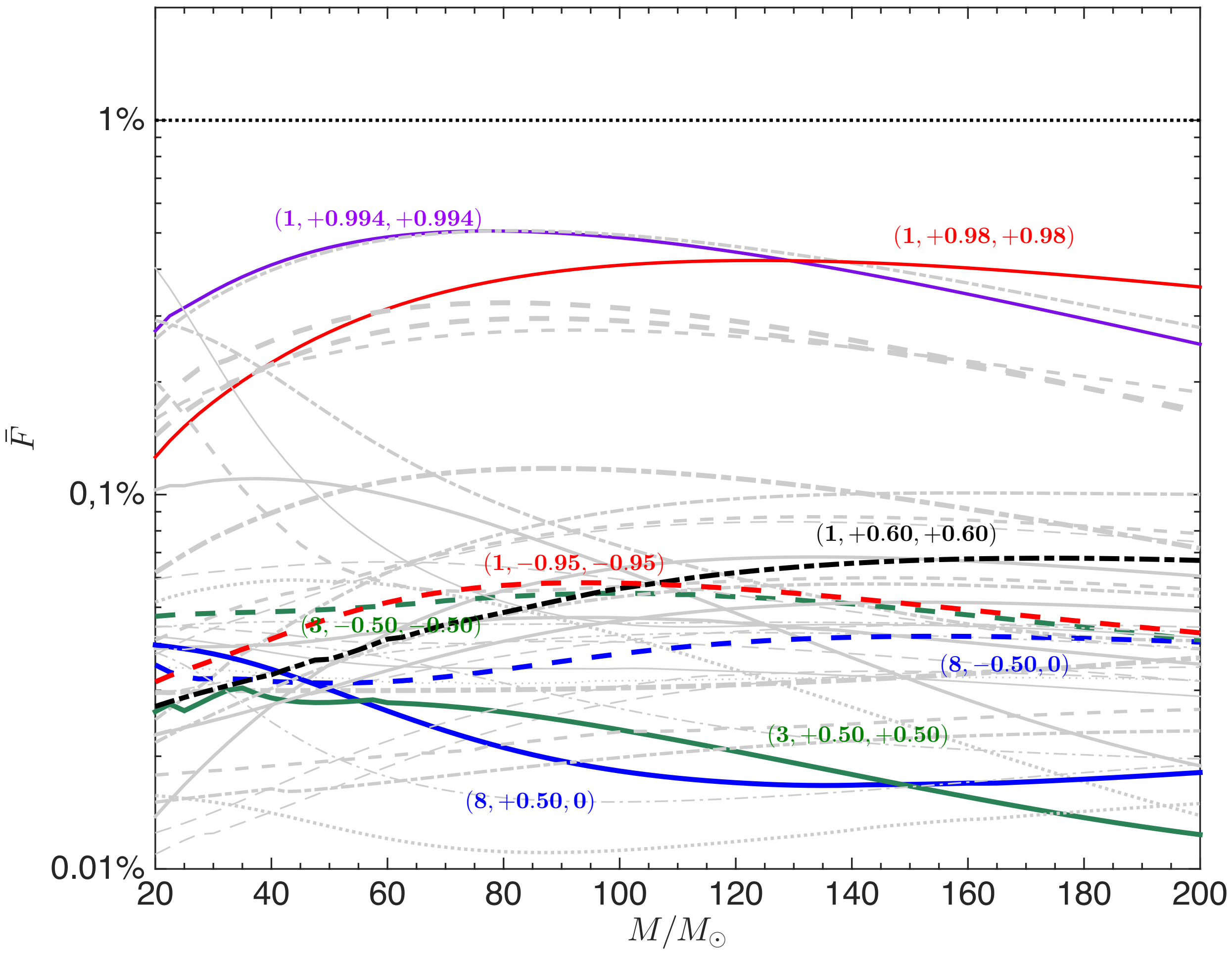
IHES EOBNR MODEL

EOB/EOBNR UNFAITHFULNESS (40 NR SXS dataset)

SEOBNRv2



IHESEOB_spin



$$\bar{F} \equiv 1 - \max_{t_0,\phi_0} \frac{\langle h_{22}^{\rm EOB}, h_{22}^{\rm NR}\rangle}{||h_{22}^{\rm EOB}||\,||h_{22}^{\rm NR}||}$$

$$\langle h_1, h_2\rangle \equiv 4\Re \int_{f_{\rm min}}^{\infty} \tilde{h}_1(f)\tilde{h}_2^*(f)/S_n(f)\, df$$

Nagar,Damour, Reisswig & Pollney, PRD 93 (2016), 044046

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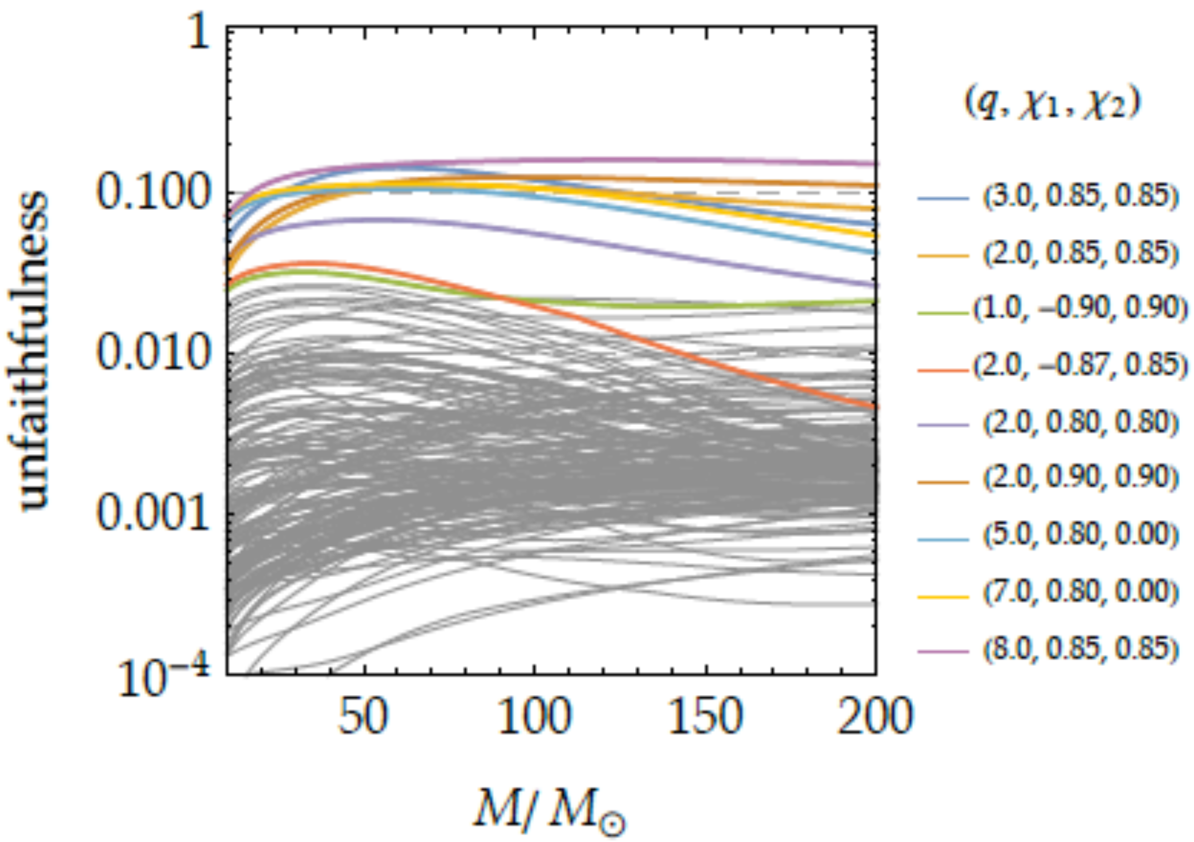
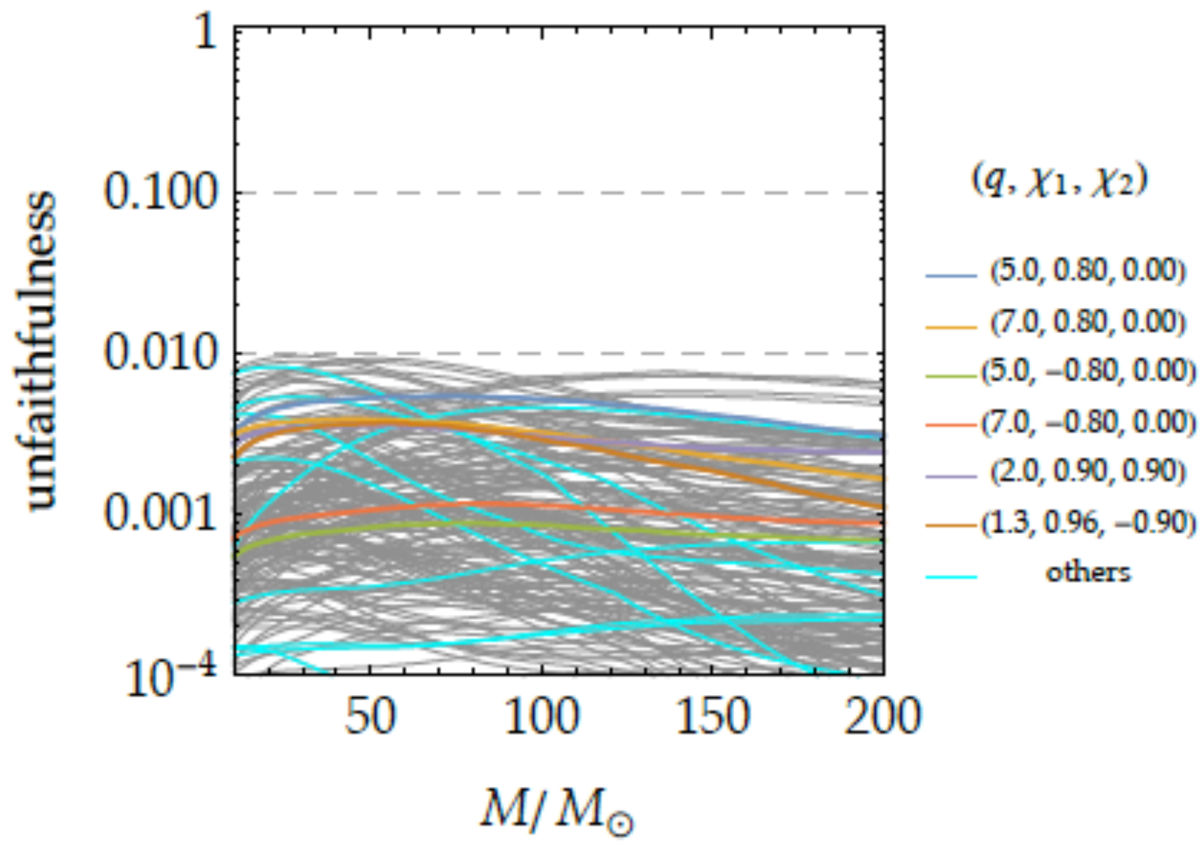
ROBUSTNESS?

SEOBNRv2



SEOBNRv4



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AEI model: Bohe et al. arXiv: 1611.03703v1

4 parameters

Strong recalibration of the state-of-the-art SEOBNRv2 model (used for O1) to have it faithful towards a set of 141 NR simulations (about 100 new ones)

$$d_{\rm SO} = +147.481449\,\chi^3\nu^2 - 568.651115\,\chi^3\nu + 66.198703\,\chi^3 - 343.313058\,\chi^2\nu + 2495.293427\,\chi\nu^2 - 44.532373\,,$$

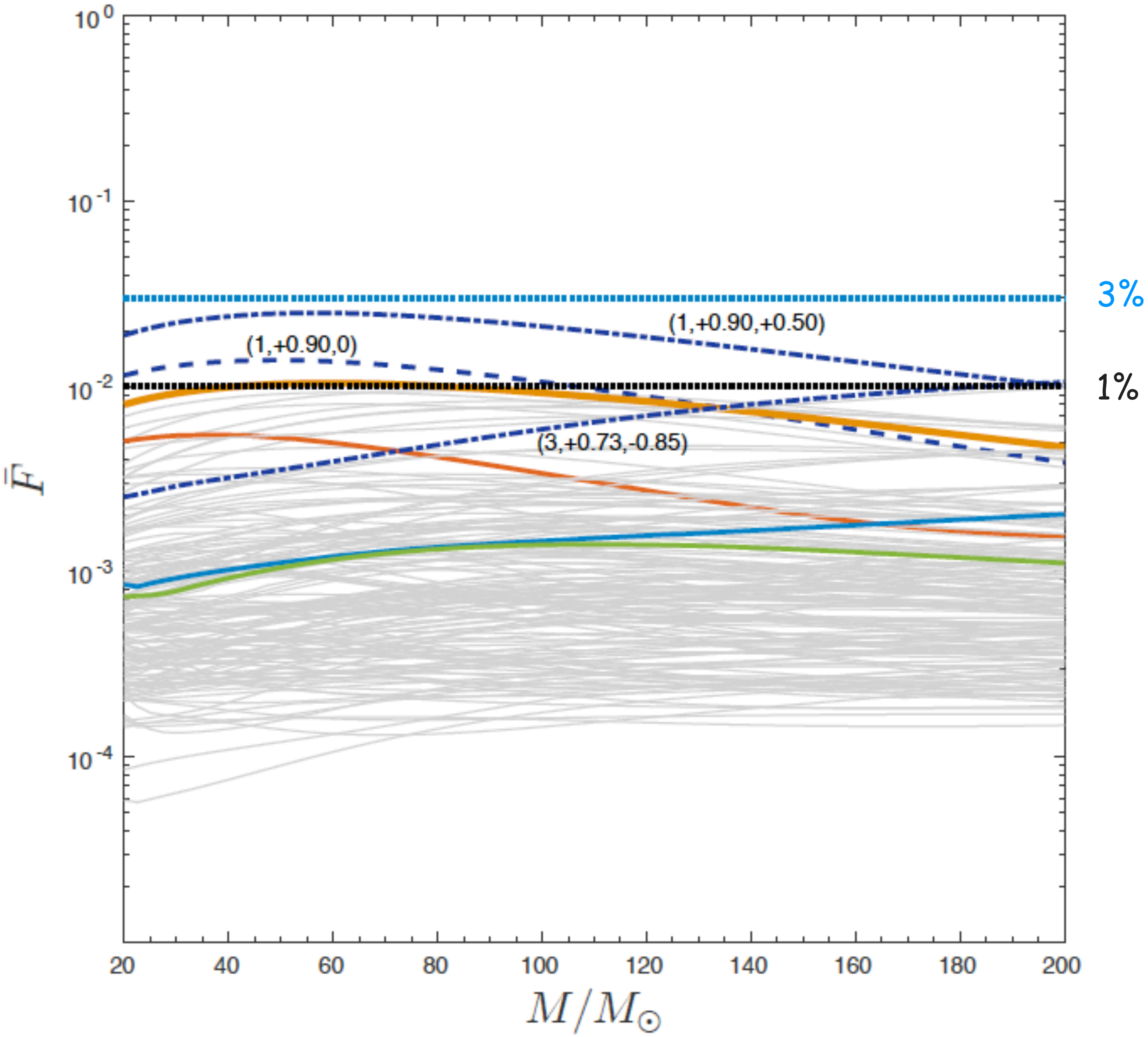
$$d_{\rm SS} = +528.511252\,\chi^3\nu^2 - 41.000256\,\chi^3\nu + 1161.780126\,\chi^2\nu^3 - 326.324859\,\chi^2\nu^2 + 37.196389\,\chi\nu + 706.958312\,\nu^3 - 36.027203\,\nu + 6.068071\,,$$

More NR simulations seem essential to “calibrate & improve” the AEI EOBNR model

BUT THIS IS NOT GENERAL...

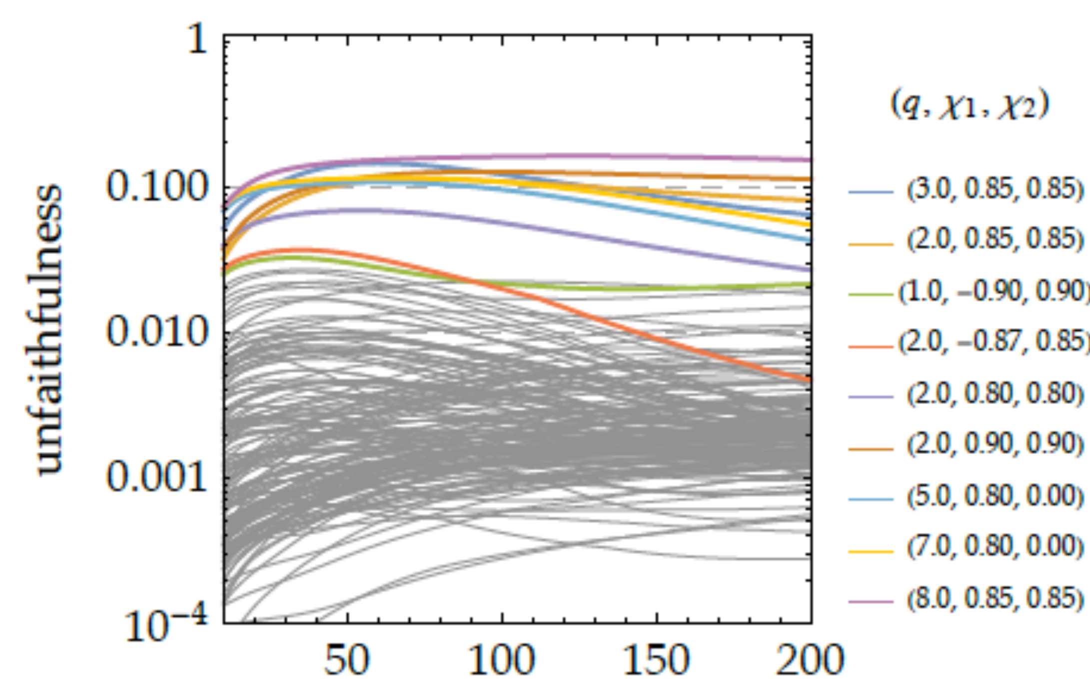
October 31st: 93 NR datasets released publicly. These are those used to calibrate SEOBNRv4 (+ others non public)
First use them to cross-check our model.

Interpolating NR fits for NQC point & ringdown. Previous NR data plus (5,-0.90,0)



Our EOBNR model is very robust and consistent ALSO outside the "information" domain over 93 new waveforms. Three outliers above 1% (but always below 3%).

Better performance than SEOBNRv2 with no need of further NR information

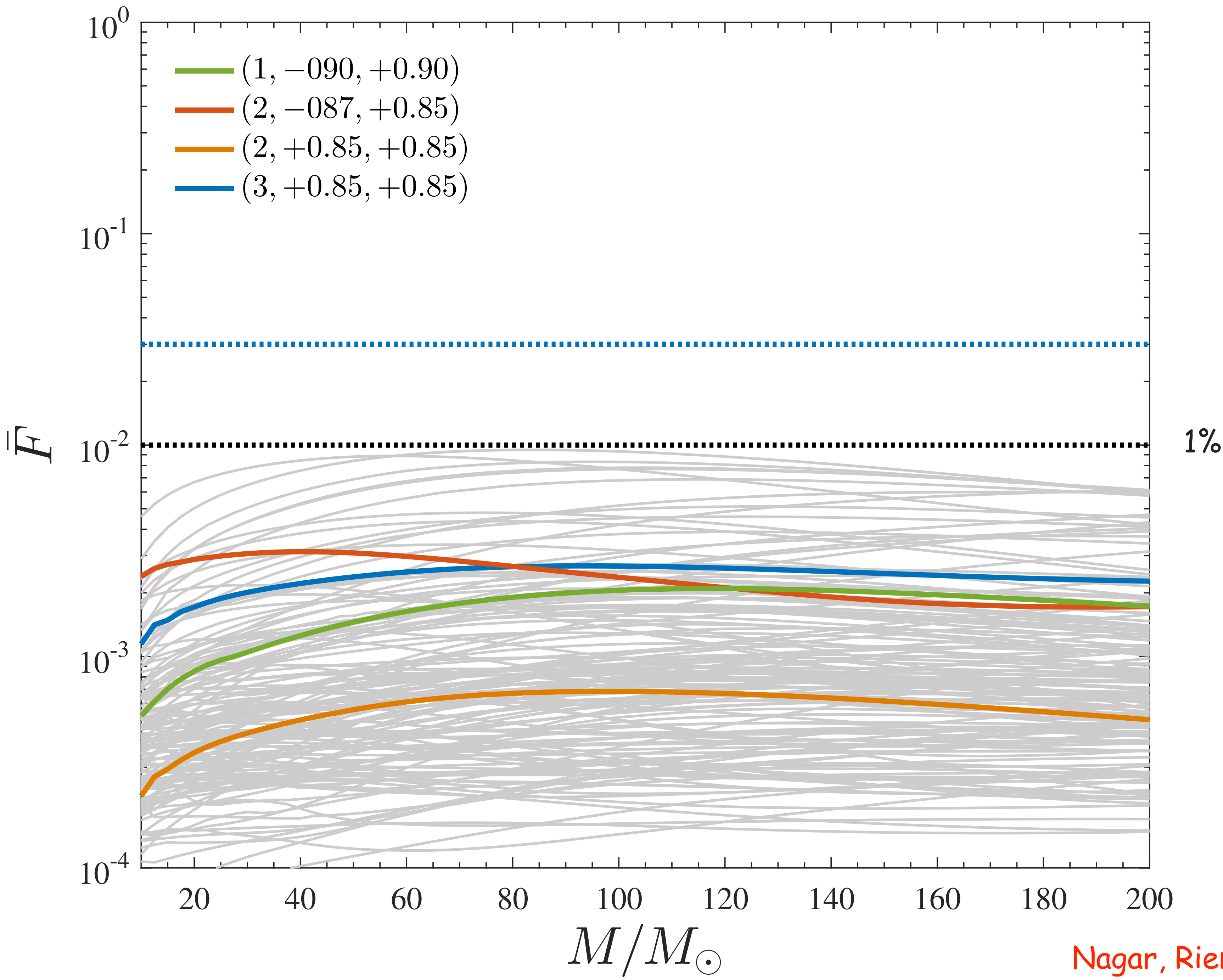


MINIMAL RECALIBRATION

Best value of the c3 parameter for the three outliers. Check phase agreement in the time-domain to be within the NR error bar. New fit to the best values to determine new values of the parameters of the unequal-mass sector.

Recalibration with 3 more NR datasets; 90 datasets as a cross/check.

Done by hand, no need of sophisticated mechanisms/algorithms. IMPROVABLE: NQC & RINGDOWN FITS USING MORE NR DATA



(3,+0.85,+0.85)
(1,+0.90,-0.90)
(2,-0.87,+0.85)
(2,+0.85,+0.85)

Nagar, Riemenschneider & Pratten, 2017, in prep.

POSTMERGER DESCRIPTION

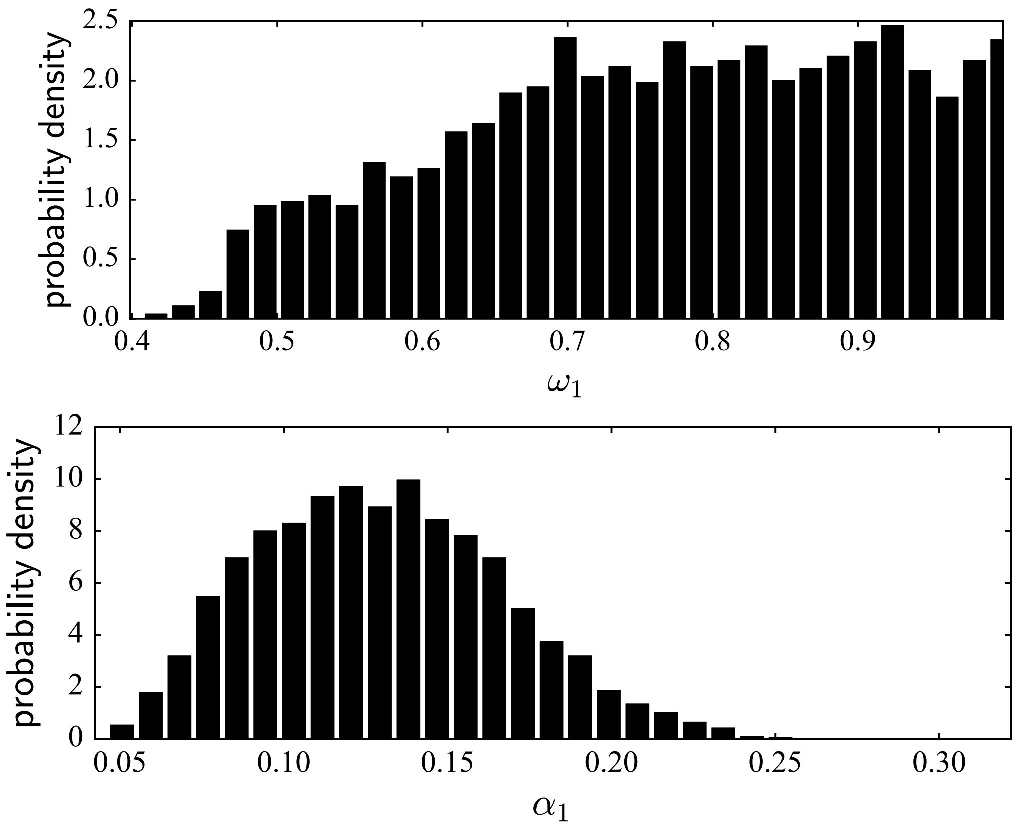
Damour&AN, PRD 2014: motivated because the "standard" QNMs attachment is far from trivial for high-spins

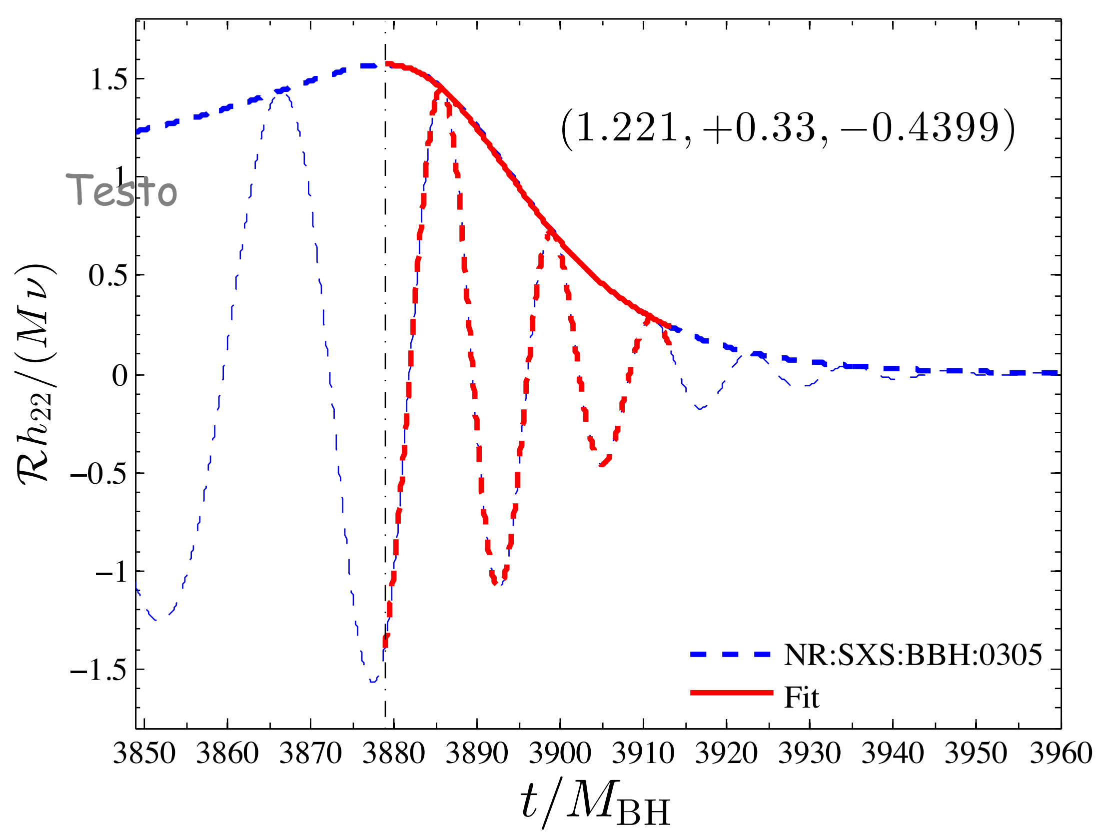
Originally conceived for EOB; useful also as a stand-alone postmerger template

Del Pozzo & AN, arXiv: 1606.03952

ANALYTIC TEMPLATE for the FULL POSTMERGER signal coming from a suitable fit of NR data.

$$\sigma_1 = \alpha_1 + i\omega_1$$





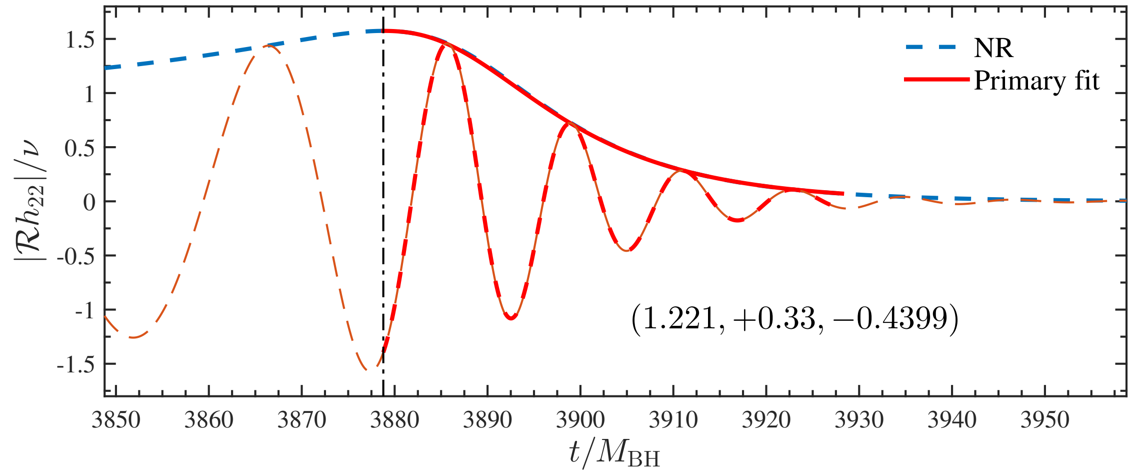
EFFECTIVE FIT

Damour&AN 2014

Factorize the fundamental QNM, fit what remains

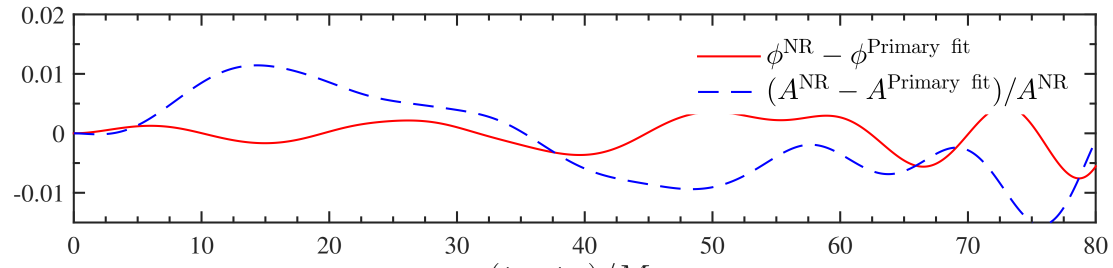
$$h(\tau) = e^{\sigma_1 \tau - i\phi_0} \bar{h}(\tau)$$

$$\bar{h}(\tau) \equiv A_{\bar{h}} e^{i\phi_{\bar{h}}(\tau)}.$$



$$A_{\bar{h}}(\tau) = c_1^A \tanh(c_2^A \tau + c_3^A) + c_4^A,$$

$$\phi_{\bar{h}}(\tau) = -c_1^\phi \ln\left(\frac{1 + c_3^\phi e^{-c_2^\phi \tau} + c_4^\phi e^{-2c_2^\phi \tau}}{1 + c_3^\phi + c_4^\phi}\right)$$



Good performance of primary fits (modulo details...)

Do this for various SXS dataset and then build up a (simple-minded) interpolating fit

$$c_2^A = \frac{1}{2}\alpha_{21}, \qquad \alpha_{21} = \alpha_2 - \alpha_1$$

$$c_4^A = \hat{A}_{22}^{\rm mrg} - c_1^A \tanh(c_3^A),$$

$$c_1^A = \hat{A}_{22}^{\rm mrg}\, \alpha_1 \frac{\cosh^2(c_3^A)}{c_2^A},$$

$$c_1^\phi = \Delta\omega \frac{1 + c_3^\phi + c_4^\phi}{c_2^\phi(c_3^\phi + 2c_4^\phi)}, \qquad \Delta\omega \equiv \omega_1 - M_{\rm BH}\omega_{22}^{\rm mrg}$$

$$c_2^\phi = \alpha_{21},$$

Black-list:

(1) the structure due to m<0 modes is not included (yet)

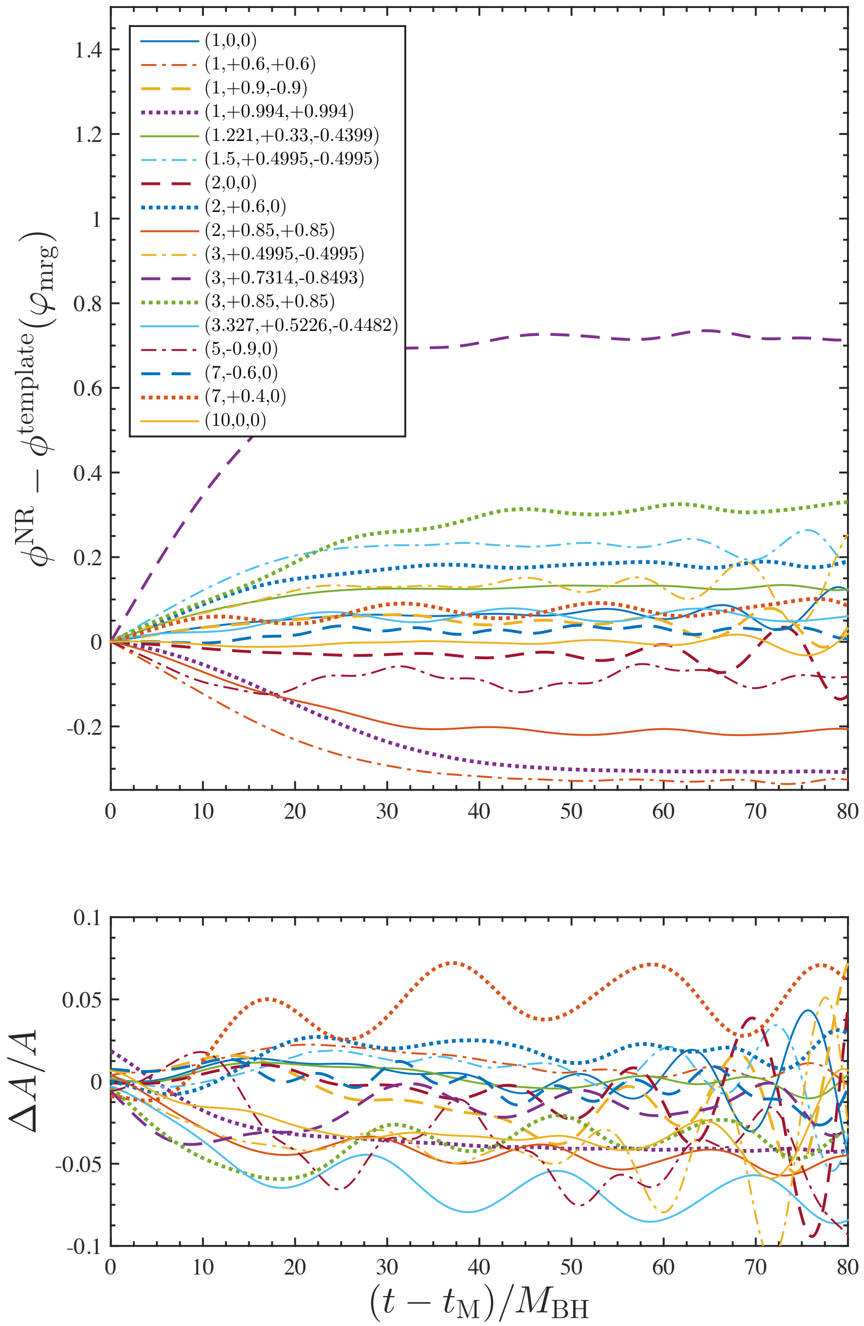
(2) large-mass ratios/high spin: amplitude problems

(3) problems are extreme for high-spin EMRL waves

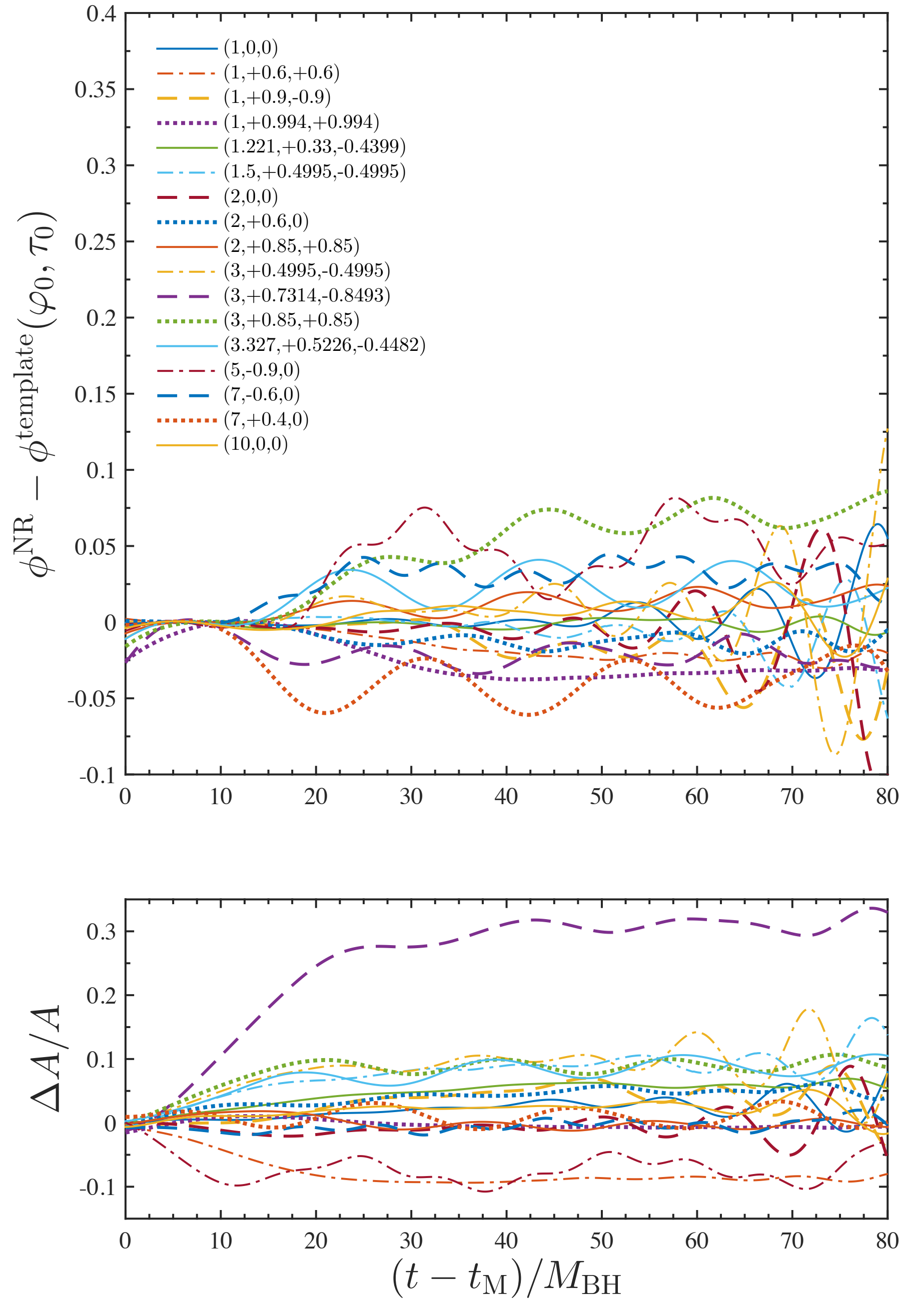
(4) more flexible fit-template needed

(5) improve/check over all datasets (SXS & BAM for large mass-ratios & consistency with EMRL)

TESTS



Phase alignment@mrg



Time&phase shift alignment (as template)

WAVEFORM RECONSTRUCTION

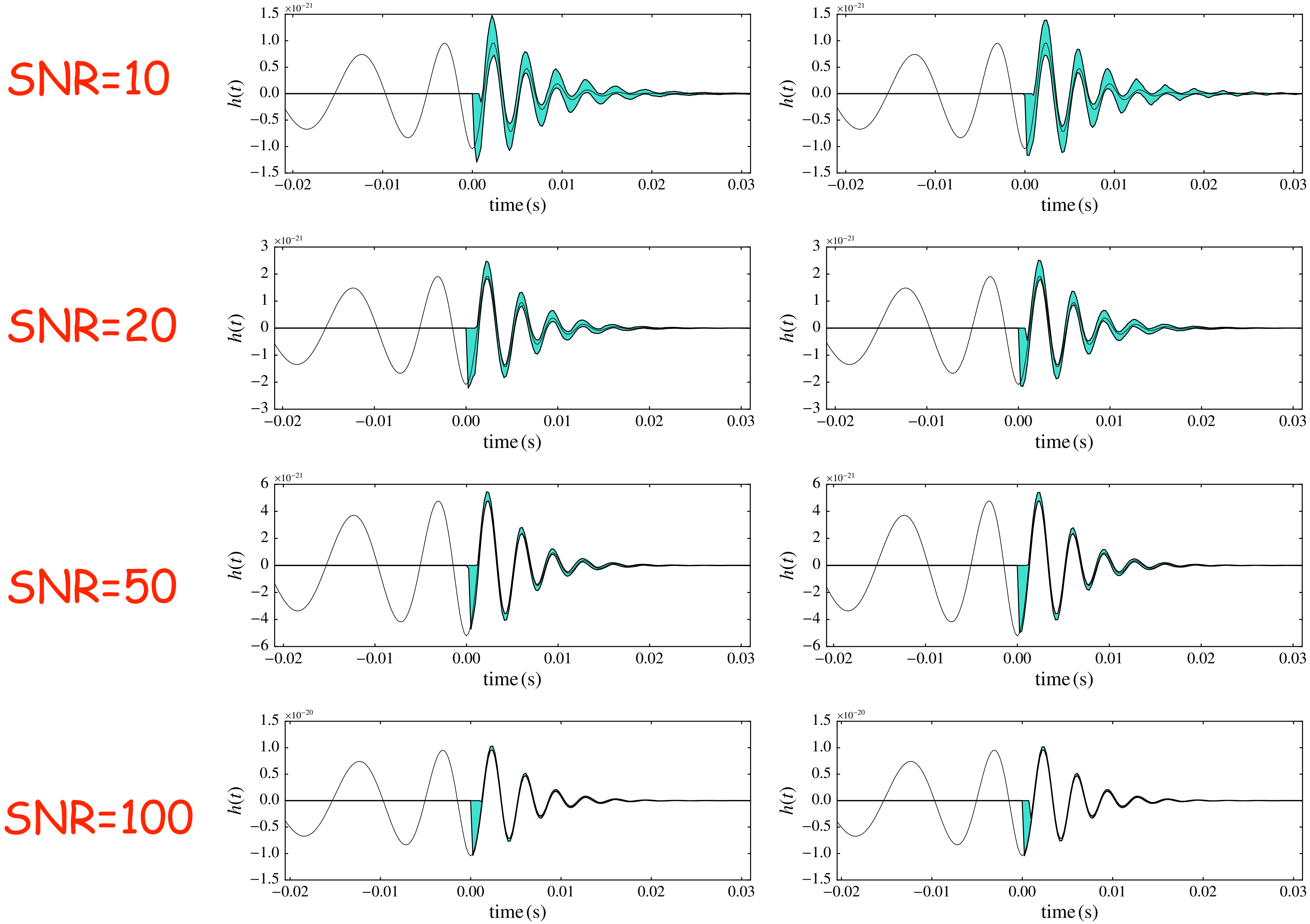
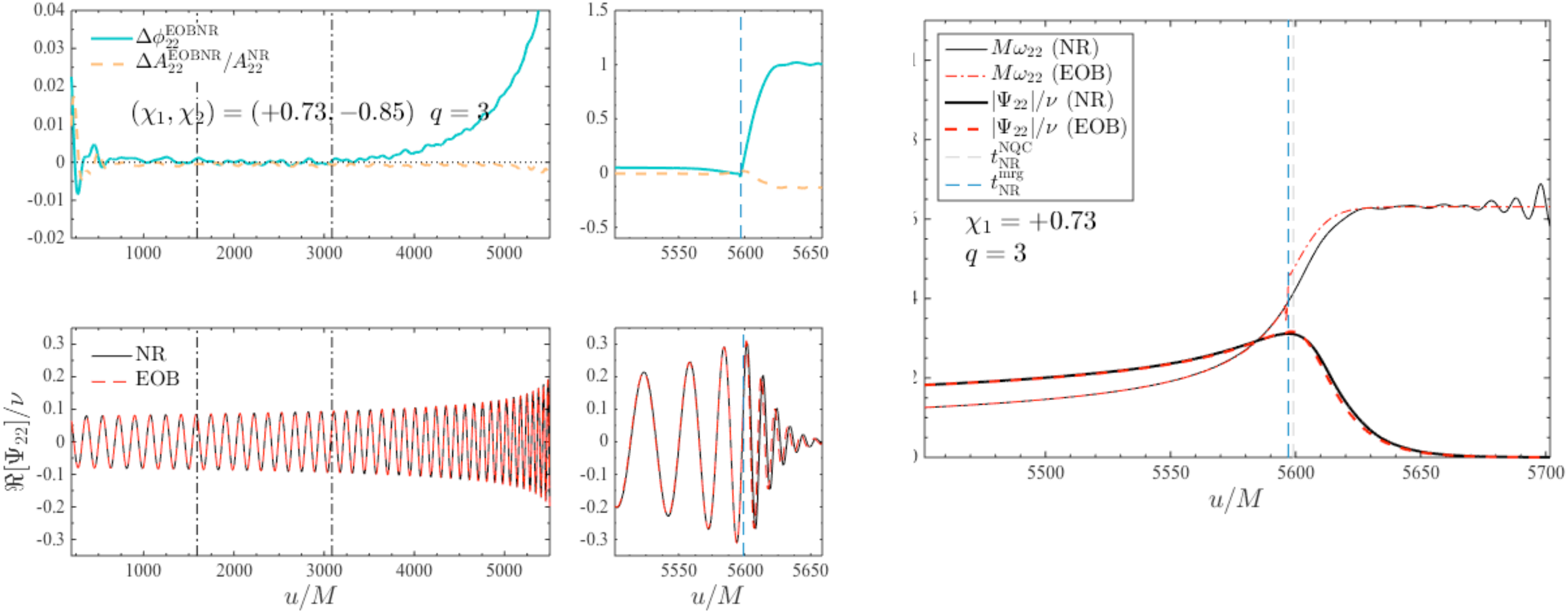


FIG. 4: From top to bottom right panels: GE case reconstructed post-merger waveform and corresponding 90% confidence region for SXS:BBH:0305 with post-merger SNR = 10, 20, 50 and 100. On the left hand side CO reconstructed post-merger waveform and corresponding 90% confidence region for SXS:BBH:0305 with post-merger SNR = 10, 20, 50 and 100. In all cases, the post-merger waveform is reconstructed very accurately, with uncertainty decreasing as the post-merger SNR increases.

WHAT TO IMPROVE?



More NR data sets to be included both in the NQC-functioning-point fit as well as in the postmerger fit (see Del Pozzo & Nagar). This is an easily solvable problem (in progress).

It is reasonable to aim at 0.1% level unfaithfulness. This is easily at reach of the model.
More precise “calibration” and/or improved theoretical structures.