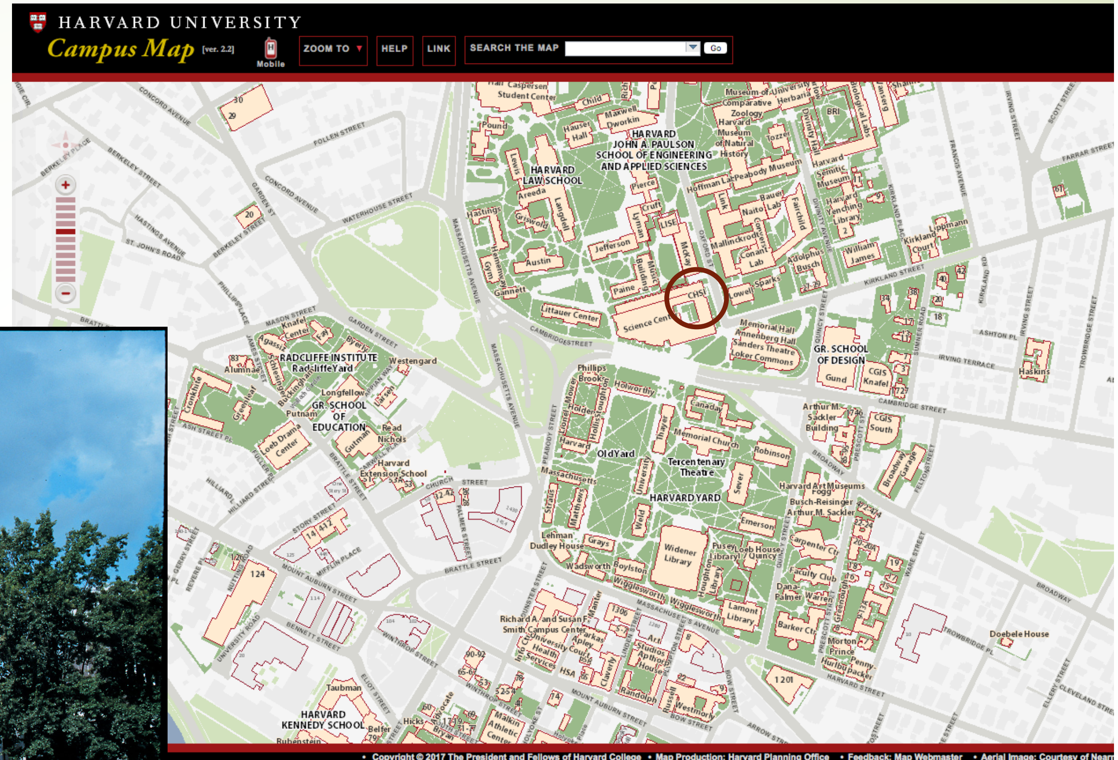
PLAYING WITH QUANTUM TOYS:

Introducing the history of quantum mechanics with historical objects

Jean-François Gauvin, PhD
Director of Administration / Lecture
Collection of Historical Scientific Instruments
Harvard University

Collection of Historical Scientific Instruments
HARVARD UNIVERSITY

Who we are

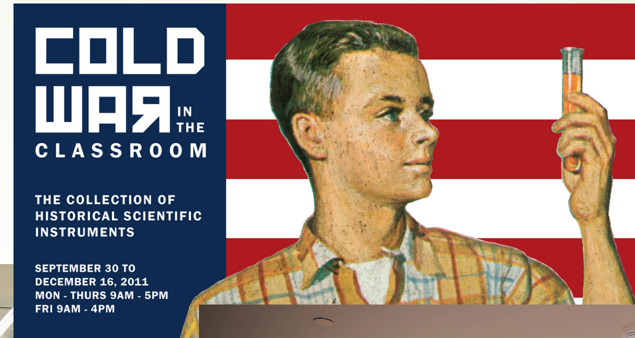


Opened Sunday to Friday, 11am to 4pm

Admission free

Who we are


ASTRONOMER'S TIME


COLD WAR IN THE CLASSROOM
THE COLLECTION OF HISTORICAL SCIENTIFIC INSTRUMENTS
SEPTEMBER 30 TO DECEMBER 16, 2011
MON - THURS 9AM - 5PM
FRI 9AM - 4PM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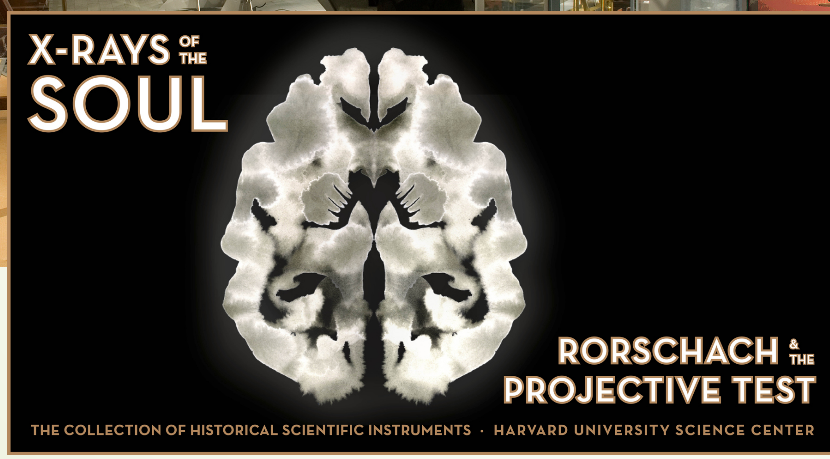
X-RAYS OF THE SOUL
RORSCHACH & THE PROJECTIVE TEST
THE COLLECTION OF HISTORICAL SCIENTIFIC INSTRUMENTS · HARVARD UNIVERSITY SCIENCE CENTER

Who we are

- ❑ Over 20,000 instruments (1400-2000) dealing with the “hard sciences”
- ❑ Over 6,000 books and archival materials

Who we are

Joseph Pope Grand Orrery, Boston, 1776-1787

MISSION STATEMENT

Through its lively exhibit and teaching programs, web presence, and increasing involvement in critical media practices, the CHSI's research activities and cultural initiatives intersect and bring together a multiplicity of academic disciplines and areas of professional museum expertise. The CHSI is both a specialized institution and an experimental space, where Harvard Faculty and students, instrument scholars and museum experts meet in the production of object-based knowledge.

Founded by David P. Wheatland (1898-1993)
(with historian of science I. Bernard Cohen)

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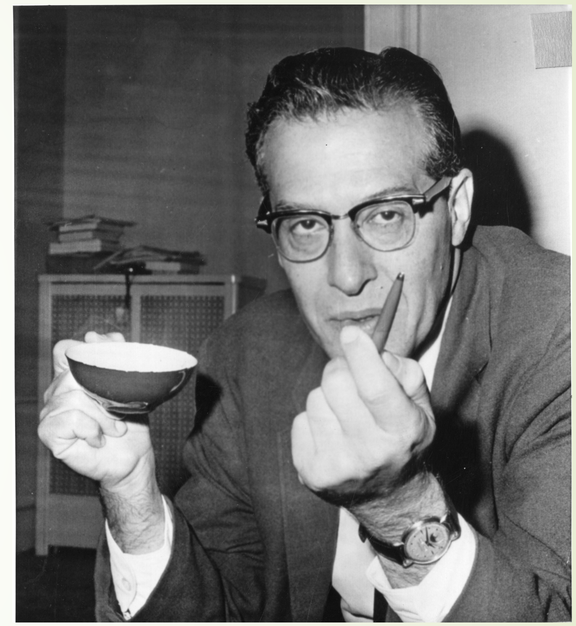
- 5 permanent staff + 4 part-time
- Students + interns (3-5 per year)
  - Working on exhibitions
  - Object and Library cataloguing

Quantum toys, aka Pauli Cubes

The strange objects were delivered to the CHSI in a package simply identified as **"Julian Schwinger, Phys 251a."**

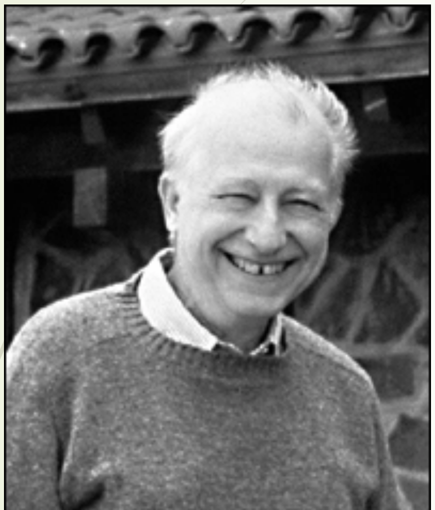


Julian Schwinger (1918-1994)

The first Albert Einstein Award medal (1951)

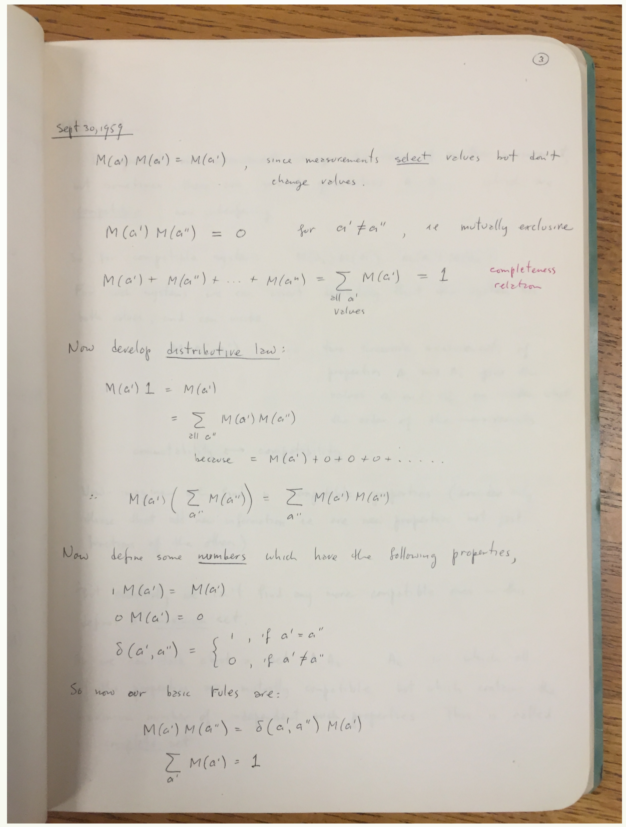
Shared with mathematician Kurt Gödel

Schwinger’s spin ½ formalism



Costas “Cos” Papaliolios
(1931-2002)

Harvard PhD in physics
(1965)



3

Sept 30, 1959

$M(a')\,M(a') = M(a')$, since measurements select values but don't change values.

$M(a')\,M(a'') = 0$ for $a' \neq a''$, ie mutually exclusive

$M(a') + M(a'') + \ldots + M(a^n) = \sum_{\text{all } a' \text{ values}} M(a') = 1$ completeness relation

Now develop distributive law:

$M(a')\,1 = M(a')$

$= \sum_{\text{all } a''} M(a')\,M(a'')$

because $= M(a') + 0 + 0 + 0 + \ldots\ldots$

$\therefore \quad M(a')\left(\sum_{a''} M(a'')\right) = \sum_{a''} M(a')\,M(a'')$

Now define some numbers which have the following properties,

$1\,M(a') = M(a')$

$0\,M(a') = 0$

$\delta(a', a'') = \begin{cases} 1, & \text{if } a' = a'' \\ 0, & \text{if } a' \neq a'' \end{cases}$

So now our basic rules are:

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$\sum_{a'} M(a') = 1$

Harvard Archives, Papaliolios notebook for Phys 251a.

On the very first day Cos was exposed to the two core prolegomena introducing Schwinger’s formalism: 1) the course was a “General approach to quantum mechanics – not historical”; 2) “The symbolic representation of the laws of atomic measurement. Stern-Gerlach experiment—measures magnetic moment”.

Schwinger’s spin ½ formalism

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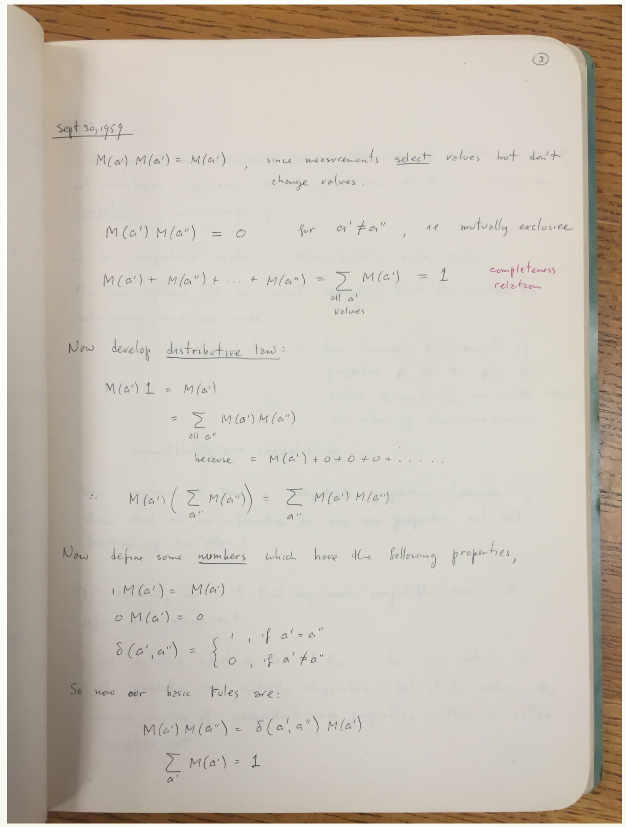

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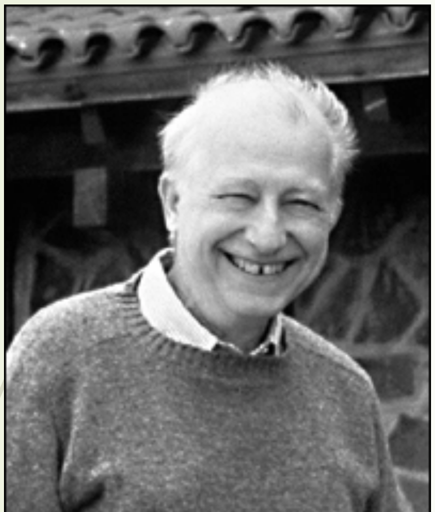

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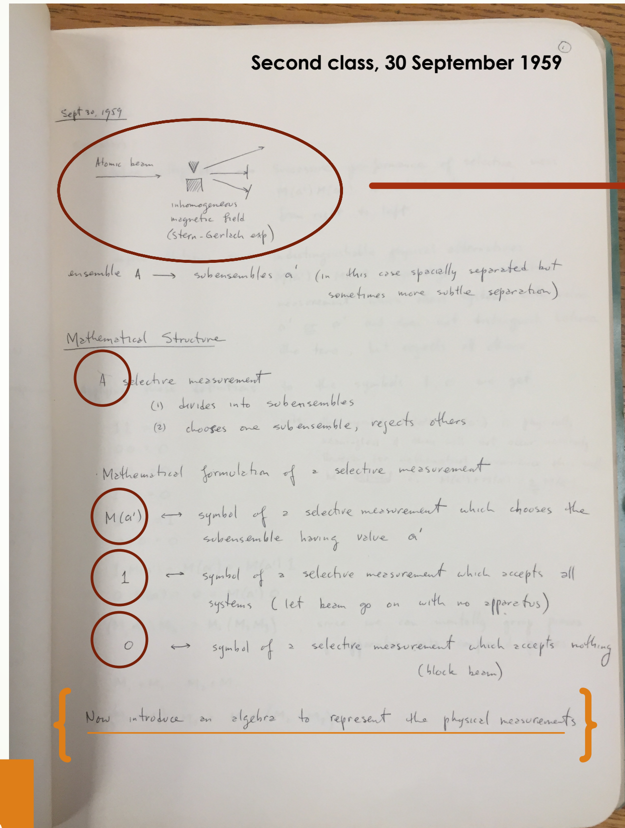
Harvard PhD in physics (1965)

« Now introduce an algebra to represent the physical measurements. »



Second class, 30 September 1959

Sept 30, 1959

ensemble A ⟶ subensembles a′ (in this case spacially separated but sometimes more subtle separation)

Mathematical Structure

A selective measurement
(1) divides into subensembles
(2) chooses one subensemble, rejects others

Mathematical formulation of a selective measurement

M(a′) ⟷ symbol of a selective measurement which chooses the subensemble having value a′

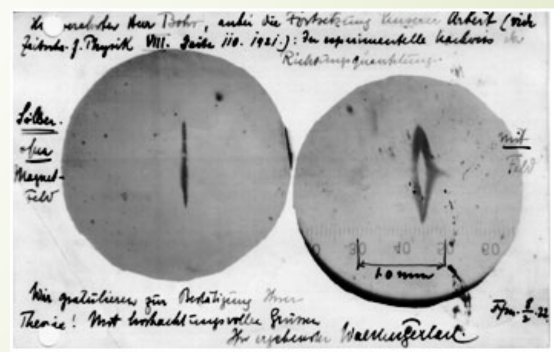
1 ⟷ symbol of a selective measurement which accepts all systems (let beam go on with no apparatus)

0 ⟷ symbol of a selective measurement which accepts nothing (block beam)

Now introduce an algebra to represent the physical measurements

Harvard Archives, Papaliolios notebook for Phys 251a.

Commemorative plaque found at the left-hand side of the entrance to the old Physikalischer Verein building in Frankfurt am Main, where Stern and Gerlach conducted their experiments. From Wikimedia Commons



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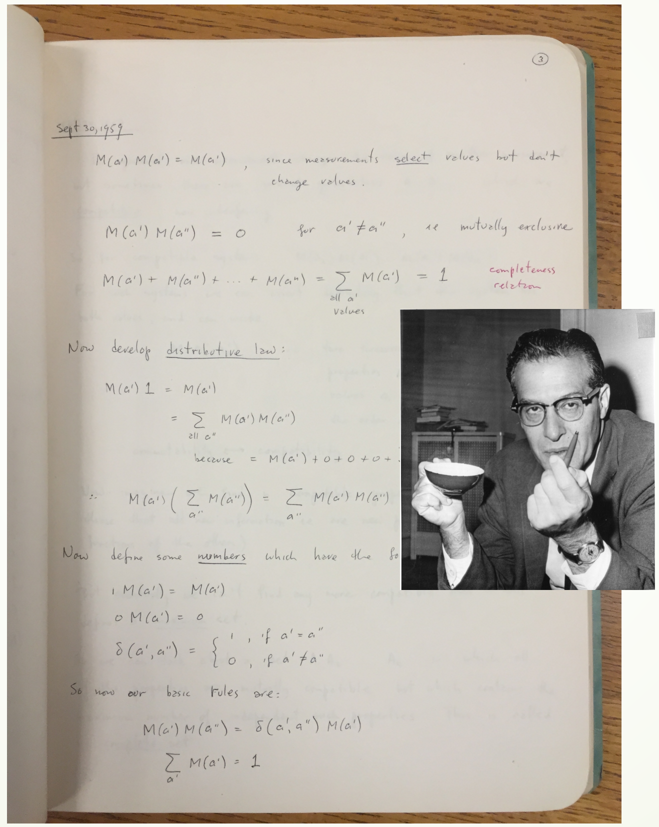

Sept 30, 1959

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Harvard Archives, Papaliolios notebook for Phys 251a.

There are thus three fundamental algebraic principles governing Schwinger's measurement symbols:

i. ***Reproducibility***, i.e. if a certain measurement is followed by a second measurement of the same property then the results of the previous measurement are repeated: M(a',a') M(a',a') = M(a',a').

ii. ***Exclusiveness***, i.e. if we make a measurement of the property A and select for the sub-assembly characterized by a', and then make a measurement upon this sub-assembly that should result in a'' (a'' ≠ a') then we will not find any such systems: M(a',a') M(a'',a'') = 0.

iii. ***Completeness***, i.e. if we look for all possible values of A, every system in the assembly will fall somewhere in that classification: $\sum_a{}^k$ M(a^k, a^k) = 1, where 1 stands for the measurement process that selects all systems.

Schwinger, *Quantum Dynamics, Part 1*, typewritten manuscript (Université de Grenoble: Les Houches, 1955), p. 2.

Schwinger's spin ½ formalism

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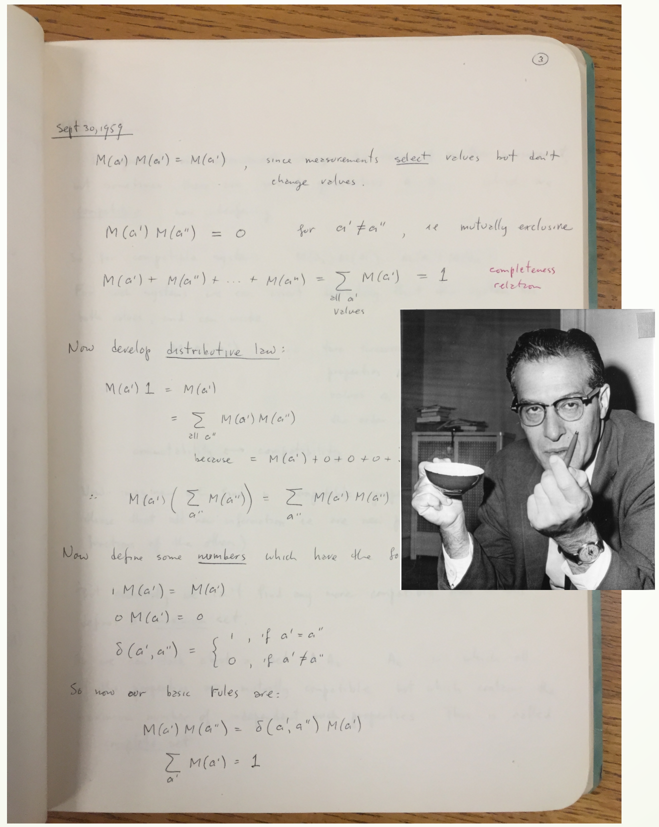

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Schwinger's primary goal was unambiguous: he "wanted to catch [the students] young and give them my version of quantum mechanics."

"My approach to teaching quantum mechanics was quite special. I would begin with a very definite approach in which quantum mechanics was a symbolism of atomic measurements. Then I would introduce a symbolism of simple Stern-Gerlach experiments, composite Stern-Gerlach experiments, symbolize it by what I called then a measurement symbol, and the measurement symbolic algebra then evolved into quantum mechanics. The spirit was just to evolve in a natural way. Not deduce, but evolve the whole machinery from the beginning. Each time I did it, it became a little more sophisticated. I was rapidly transforming quantum mechanics into my own image."

Quoted from Mehra and Milton (2000), p. 156.

Schwinger's spin ½ formalism

In the Fall of 1950, a young **Jeremy Bernstein**, then a Senior at Harvard, enrolled in Schwinger's graduate student course of advanced quantum mechanics, Physics 253. He discovered what many other students would over the next three decades:

"To say that Schwinger's lectures were both brilliant and impenetrable would be an understatement. They were very brilliant and impenetrable. **Schwinger was, it turned out, trying out an entirely new formulation of the theory on us—the old one would have been hard enough**—and since he lectured from memory questions were discouraged. … After a few weeks I was lost. I was commiserating with a friend, a physical chemist, who was also taking Schwinger's course. He said, what you should do is come with me to MIT and audit "Viki." Viki, I learned, was Victor Weisskopf, who was Schwinger's analogue—I am tempted to say antiparticle—at MIT."

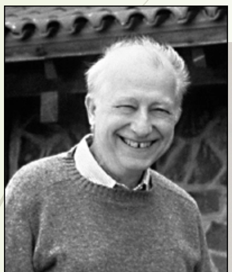
Jeremy Bernstein, "The Charms of a Physicists," *The New York Review of Books*, April 11, 1991.

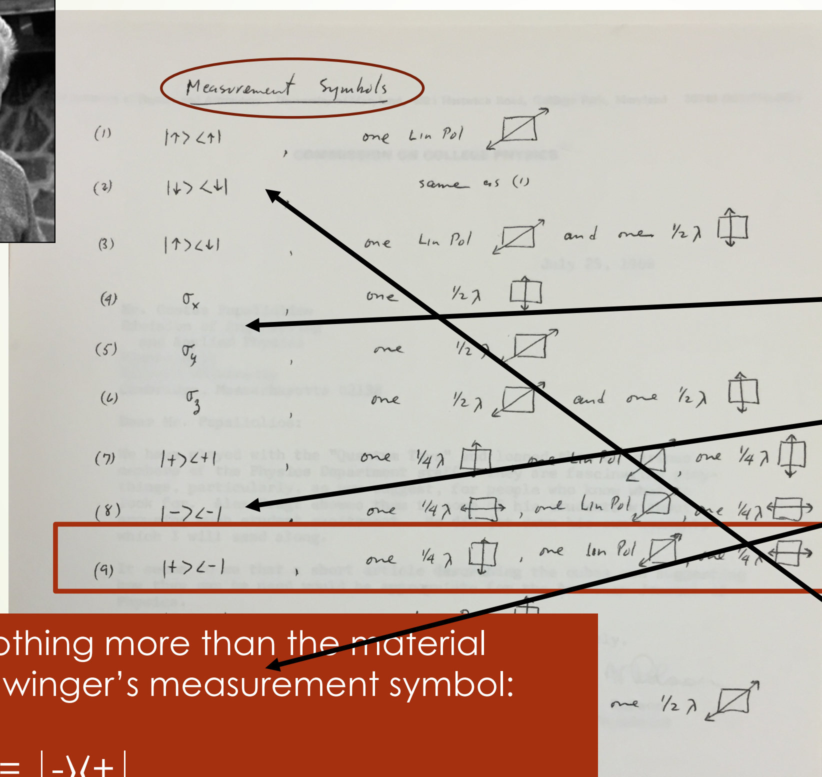
Even Richard Feynman, the ever so flamboyant and more famous physicist in the public eye, was struck by Schwinger's masterful presentations when the latter visited Los Alamos in 1945:

"I'm not sure exactly the subject, but it was a scene that you probably have seen once. The beauty of one of his lectures. He comes in, with his head a little bit to one side. He comes in like a bull into a ring and puts his notebook down and then begins. And the beautiful, organized way of putting one idea after the other. Everything very clear from the beginning to the end. You can imagine for a lecturer like me, what a sensation it was to see such a thing. I was supposed to be a good lecturer according to some people, but this was really a masterpiece."

[Quoted from Schweber (1994), p. 300.]

Another familiar 2-state system: polarized light





Measurement Symbols

(1) $|\uparrow\rangle\langle\uparrow|$, one Lin Pol

(2) $|\downarrow\rangle\langle\downarrow|$ same as (1)

(3) $|\uparrow\rangle\langle\downarrow|$, one Lin Pol and one $\frac{1}{2}\lambda$

(4) σ_x , one $\frac{1}{2}\lambda$

(5) σ_y , one $\frac{1}{2}\lambda$

(6) σ_z , one $\frac{1}{2}\lambda$ and one $\frac{1}{2}\lambda$

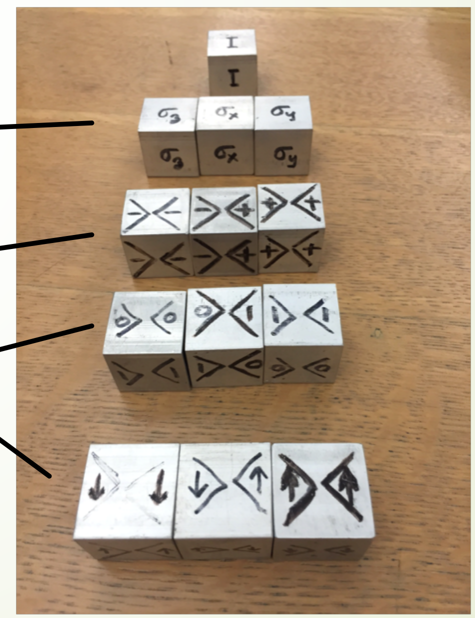
(7) $|+\rangle\langle+|$, one $\frac{1}{4}\lambda$, one Lin Pol , one $\frac{1}{4}\lambda$

(8) $|-\rangle\langle-|$, one $\frac{1}{4}\lambda$, one Lin Pol , one $\frac{1}{4}\lambda$

(9) $|+\rangle\langle-|$, one $\frac{1}{4}\lambda$, one Lin Pol , one $\frac{1}{4}\lambda$

one $\frac{1}{2}\lambda$

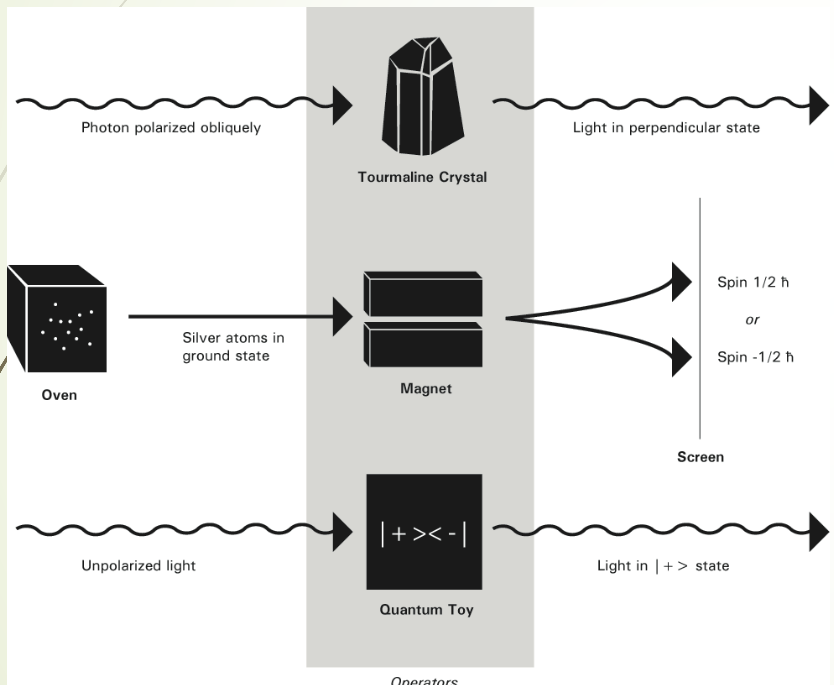
Most likely Polaroid filters HN32



The cube |-⟩⟨+| is nothing more than the material embodiment of Schwinger's measurement symbol:

M(a',a'') = M(-,+) = |-⟩⟨+|.

Reifying the notion of “operators”



The top image schematizes the polarization experiment with a tourmaline crystal; the middle image schematizes the Stern-Gerlach experiment; the bottom image represents one quantum toy’s associated arrangement. The crystal, the magnet and the quantum toys are called “operators”. Two quantum toys set side by side would be represented by two crystals and two magnets in the other two configurations. Illustration by Maureen Ton, CHSI

“In teaching the concept of linear operators, it is particularly difficult to convey the distinction between a linear operator and its representation as a matrix. Students are confused when told that the same linear operator may be represented by different matrices depending on the choice of basis vectors. This is because they find the operator hard to visualize as an independent entity. This problem is solved by the **Pauli cubes**. One can hold a σ_x cube in his hand and say, “Here is a linear operator!” Then using the other cubes, one can find its matrix representation in various sets of bases. Matrix multiplication, and lack of commutativity are also readily demonstrable.”

How do the quantum toys work?

The light prepared in the $|\uparrow\rangle$ state (using the $|\uparrow\rangle\langle\uparrow|$ cube) could be described in terms of the superposition of the $|+\rangle$ and $|-\rangle$ states.

Based on this demonstration it is possible to introduce the proper symbolism (which mirrors the classical laws of polarization):

$|\uparrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$ and $|\downarrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$

with $\langle+|+\rangle = \langle-|-\rangle = 1$ (i.e. light comes through) and $\langle+|-\rangle = \langle-|+\rangle = 0$ (i.e. light doesn’t come through).

Simple algebra will show that $\langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 1$ and that $\langle\uparrow|\downarrow\rangle = \langle\downarrow|\uparrow\rangle = 0$

How do the quantum toys work?

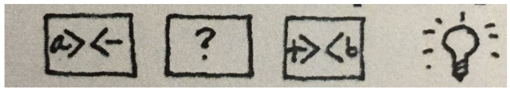
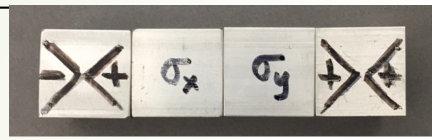
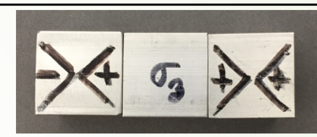
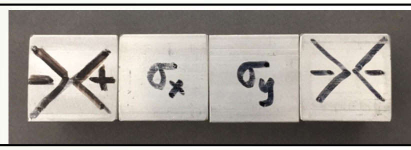
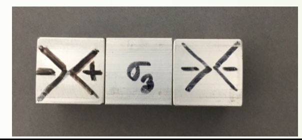
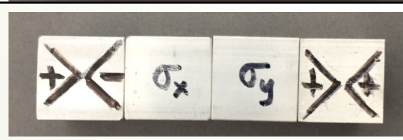
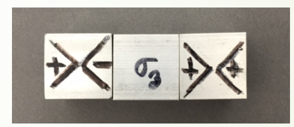
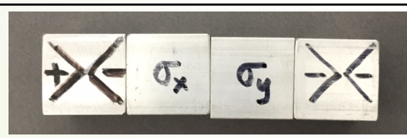
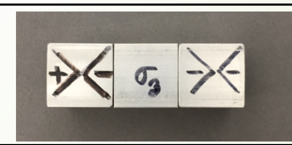
Quantum Toys	Looking through	Matrix representation
	= ?	$\sigma_x = \begin{pmatrix} ++ & +- \\ -+ & -- \end{pmatrix}$
	= 0	$\sigma_x = \begin{pmatrix} 0 & +- \\ -+ & -- \end{pmatrix}$

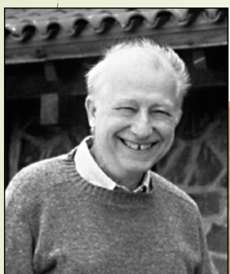
Table 1: Playing with the σ_x operator to determine its function. In this case, it switches the |+⟩ state into the |-⟩ state, and vice versa. In looking into the cubes, 0 signifies light doesn't go through; 1 signifies light goes through.

You can similarly determine the matrix representation of all the σ operators, all in ***base states***, thus demonstrating how

"…the same linear operator may be represented by different matrices depending on the choice of basis vectors."

Quantum Toys	Looking through	Equivalent to
	= 1	
	= 0	
	= 0	
	= 1	

How do the quantum toys work?



According to
principles car

1. Connectio
2. The non-cc
3. The conce
4. Dirac nota
5. Descriptior
6. Unitary Trar
7. Pauli spin operators
8. Isotopic spin
9. K mesons

……………… Etc.

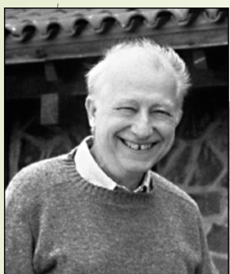
The theoretical approach put forward by Murray Gell-Mann and Abraham Pais in 1955 was not to consider K^0 and $\bar{K}^0$ as two distinct mesons, but rather as *one* two-state system. The two new "particles" or states they defined to "save the phenomena" derived from a superposition of K^0 and $\bar{K}^0$ such that

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \text{ and } |K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle),$$

which is exactly the superposition of states described above using the quantum toys!

Introducing the concept of « strangeness » recently discovered, to help understand the ambivalent decay of K mesons into matter and anti-matter.

How do the quantum toys work?



From 1962 to roughly 1998, Papaliolios “played” with his quantum toys. From the beginning they were associated to Schwinger.

Even in 1998, the table of content heading a few pages of handwritten notes intended for teaching, together with another set of 55 small cubes, makes the relationship to Schwinger even more explicit:

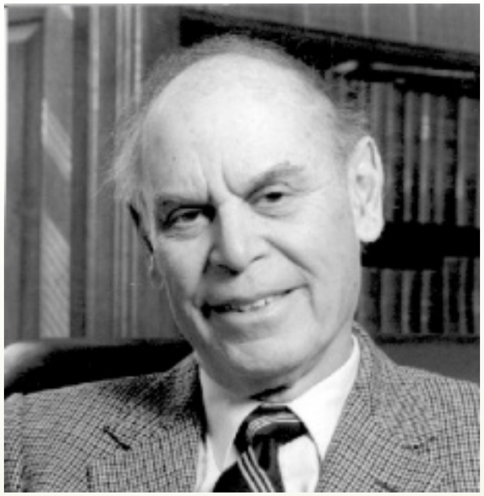
1. QM 2-state system
2. **Meas[urement] Algebra — Schwinger**
3. Use photon — <u>Polarization</u>
4. Define <u>state</u>, <u>measurement operation</u>
5. **Assign symbols to items in (4) [Schwinger’s M(a’,a’’) symbol]**

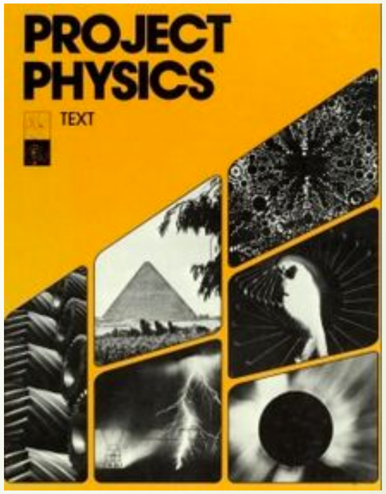
Papaliolios, manuscript titled “Notes on Measurement Symbols ’98”. Emphasis original.

Conclusion

- The Quantum Toys were Unsuccessful
- The timing of their creation, however, is interestingly connected to a Harvard initiative designed to rethink how physics was taught at the high-school level during the 1960s.

Gerald Holton, a former student of Percy Bridgman and a rising star in the field of the history and philosophy of science, was spearheading with two colleagues, F. James Rutherford and Fletcher G. Watson, the Project Physics Course—also known as the Harvard Project Physics.





Conclusion

- The Quantum Toys wer
- The timing of their crea
  Harvard initiative desig
  school level during the
- Learning by doing

In Memoriam

Silvan (Sam) Schweber (1928-2017)