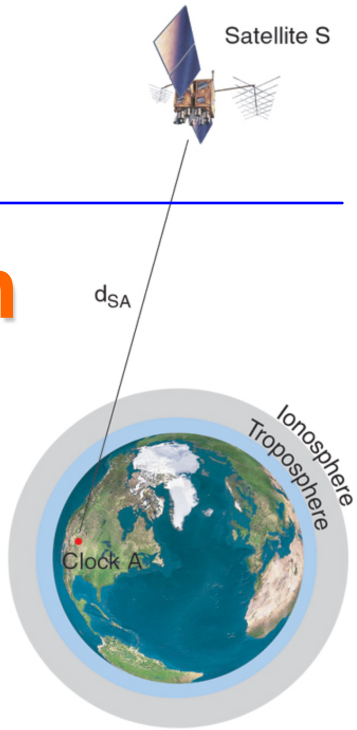
Progetto Extreme Energy Events (EEE)
La Scienza nelle Scuole

On the use of GNSS

for atomic clocks comparison



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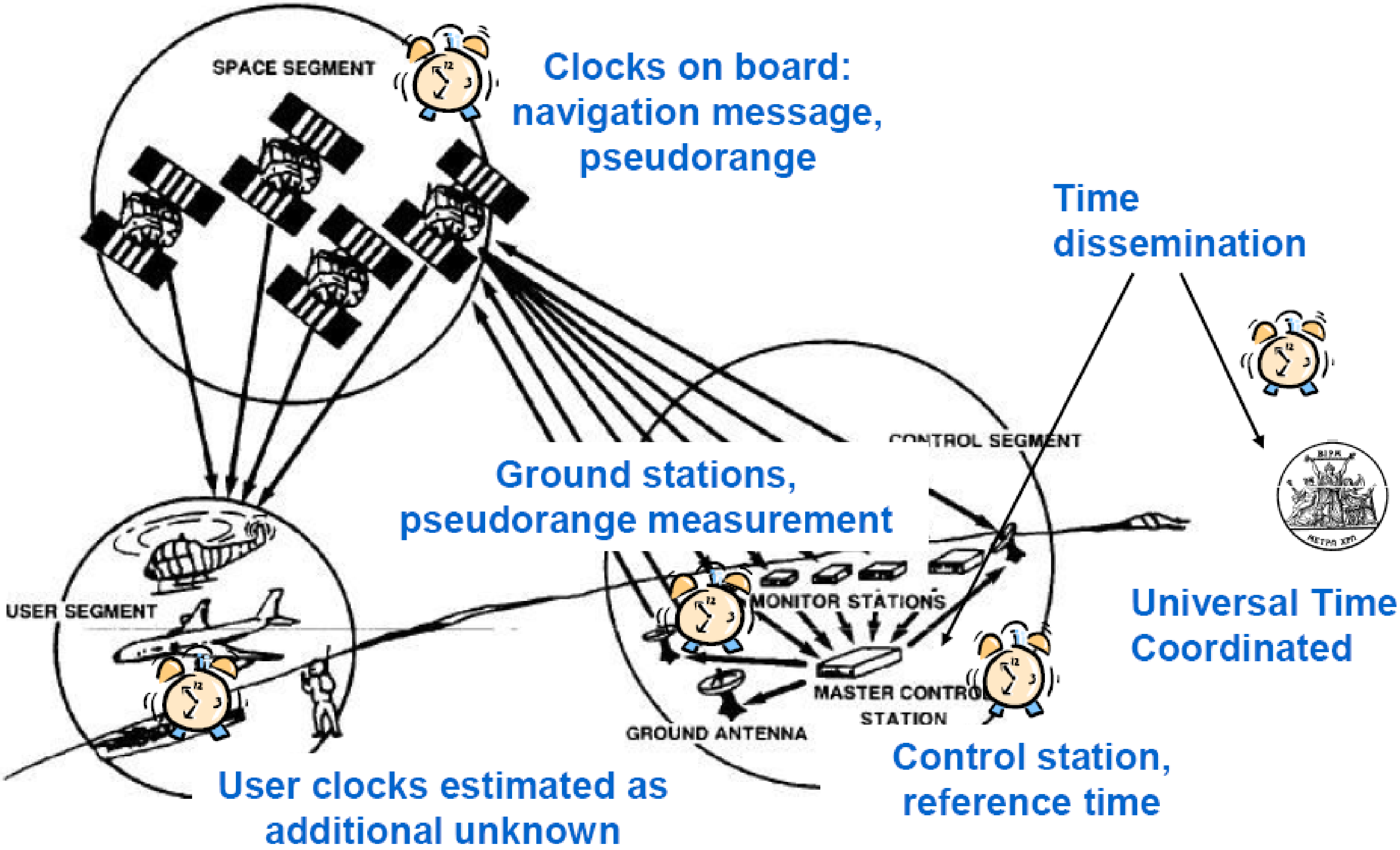
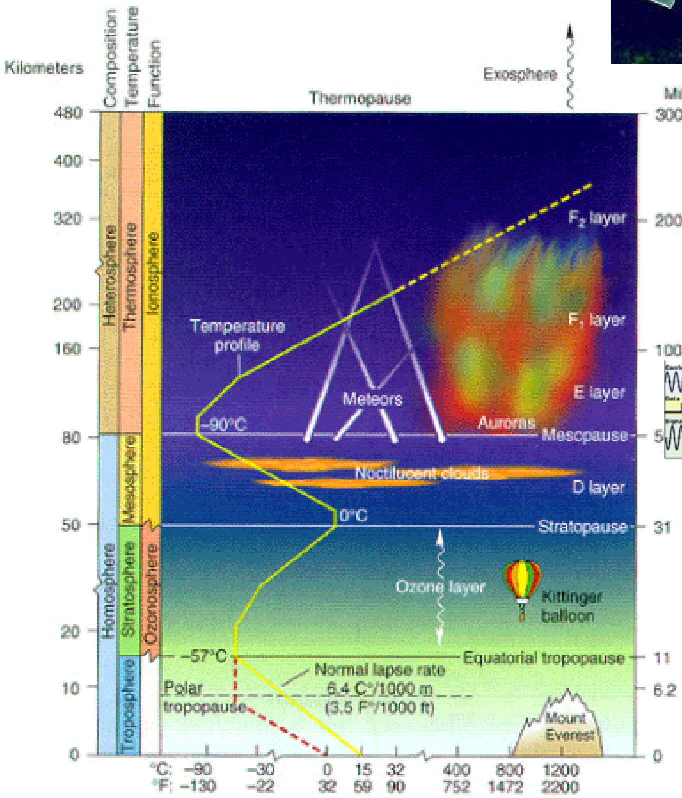
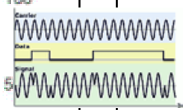
GNSS: Global Navigation Satellite System



Main leadings





- GPS satellite veichle (SV) and its on-board clock
  - Atomic frequency standards (Rb, Cs)
  - Availability of GPS Time!
- Signals-in-space (SIS) transmitted by the GPS SV
  - L1/L2 Carriers, CA/P code modulations, Navigation data
- Satellite-to-Earth propagation medium
  - Ionosphere
  - Troposphere
- GPS receiver and its antenna
  - Fixed (and known) location
  - Timing or geodetic receiver
- Internal/External frequency reference for receiver
  - Atomic clocks (H-maser, Cesium…)
  - UTC(k) time scale

1PPS

Ref. Freq

INRiM ISTITUTO NAZIONALE DI RICERCA METROLOGICA

GPS observables

- Code (pseudorange) measurement
  - Apparent transition time of the GPS modulated signal (code) from a satellite to the receiver
  - Only C/A-code on L1 (Standard Positioning Service, SPS)
    P(Y)-code on both L1 and L2 (Precise Positioning Service, PPS)
  - Biased and noisy estimate of istantaneous actual range (distance) between receiver and satellite

- Carrier phase measurement
  - Difference between the phases of the receiver-generated carrier signal and the carrier received from a satellite at the measurement epoch
  - Both L1 and L2 frequencies (available to all users)
  - Precise measurement of change in the satellite-receiver pseudorange over a time interval and estimates of its istantaneous rate (Doppler frequency)
  - Precise but ambiguous estimate of istantaneous actual range between receiver and satellite (modulo-2π measurement only)

Pseudorange (code) measurement

Basics

Based on the measurement of the apparent transition time of the GPS modulated signal (e.g. C/A code) from a satellite to the receiver

Time shift required to align the C/A-code replica generated at the receiver with the signal received from the satellite

- Each GPS satellite generates the signal in accordance with its on-board atomic clock
- The receiver generates the replica of each received signal in accordance with its clock
- The receiver aligns the C/A-code replica generated at the receiver with the signal received from the satellite
- Satellite clock and receiver clock are not synchronized ($dt_R^{(S)} \neq 0$)

Biased and noisy measurement of the actual transit time, τ

$$\tau' = (\tau + dt_R^{(S)}) + \varepsilon_\tau$$

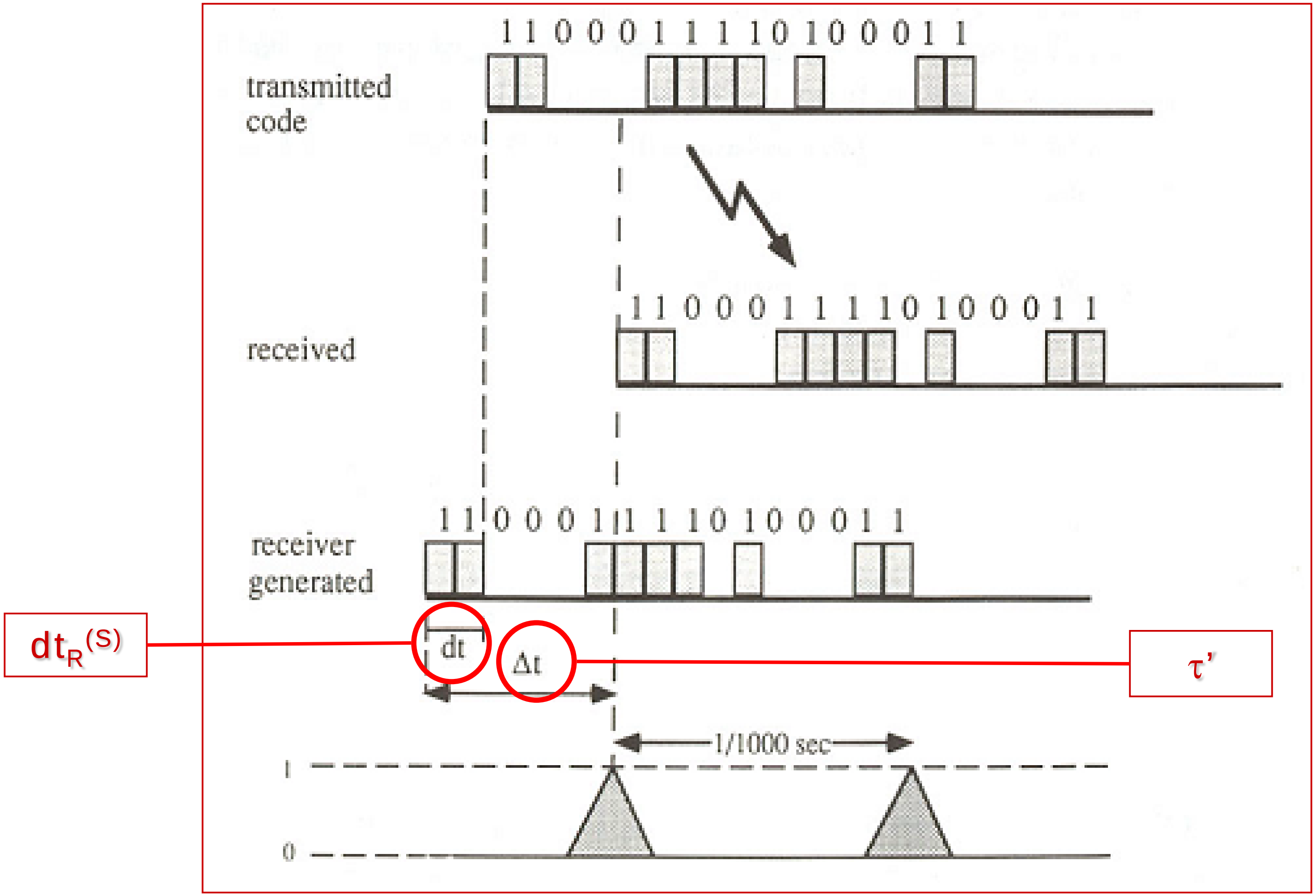
$$\Downarrow$$

Biased and noisy measurement of the actual range, $R_R^{(S)}$

$$\rho_R^{(S)} = c \cdot \tau' = c \cdot (\tau + dt_R^{(S)} + \varepsilon_\tau)$$
$$= R_R^{(S)} + c \cdot dt_R^{(S)} + c \cdot \varepsilon_\tau$$

Pseudorange (code) measurement (cont'd)

Basics (cont'd)



Pseudorange (code) measurement (cont'd)

Measurement model

Pseudorange measurement performed at epoch t (GPS Time) by the receiver using the incoming SIS transmitted at epoch $(t - \tau)$ from a single GPS satellite:

$$\rho_R^{(S)}(t) = R_R^{(S)}(t, t-\tau) + c \cdot [\delta t_R(t) - \delta t^{(S)}(t-\tau)] + I_R^{(S)}(t) + T_R^{(S)}(t) + \varepsilon_R^{(S)}(t)$$

(expressed in distance units, i.e. meters)

$R_R^{(S)}$	actual geometric range between satellite and receiver (in meters)
δt_R	receiver clock offset wrt GPS Time: $\delta t_R = t_R - t_{GPS}$ (in seconds)
$\delta t^{(S)}$	satellite clock offset wrt GPS Time: $\delta t^{(S)} = t^{(S)} - t_{GPS}$ (in seconds)
$I_R^{(S)}$	ionosphere propagation delay (in meters)
$T_R^{(S)}$	troposphere propagation delay (in meters)
$\varepsilon_R^{(S)}$	both modelling errors and unmodelled errors (e.g., multipath) (in meters)
c	vacuum speed of light (299 792 458 m/s)

How to get GPS Time?

satellite orbits

$$\rho_R^{(S)}(t) = R_R^{(S)}(t,t-\tau) + c \cdot [\delta t_R(t) - \delta t^{(S)}(t-\tau)] + I_R^{(S)}(t) + T_R^{(S)}(t) + \varepsilon_R^{(S)}(t)$$

- The fixed position of the receiver at the signal reception time t is known (from a previous survey)
  - Phase center of the receiver's antenna
  - Coordinates in ECEF (Earth-Centered Earth-Fixed) system
- The position of the GPS satellite at the signal transmission time (t-τ) can be computed by the receiver using the broadcast ephemeris (Subframes 2 and 3 of the navigation message)
  - Predicted by GPS CS for each satellite
  - Keplerian orbital parameters
  - Phase center of the SV's antenna
  - ECEF coordinates

How to get GPS Time? (cont'd)

satellite clock

$$\rho_R^{(S)}(t) = R_R^{(S)}(t,t-\tau) + c \cdot [\delta t_R(t) - \delta t^{(S)}(t-\tau)] + I_R^{(S)}(t) + T_R^{(S)}(t) + \varepsilon_R^{(S)}(t)$$

- Predicted by the GPS Control Segment for each satellite
- Modeled as a quadratic function over a time interval

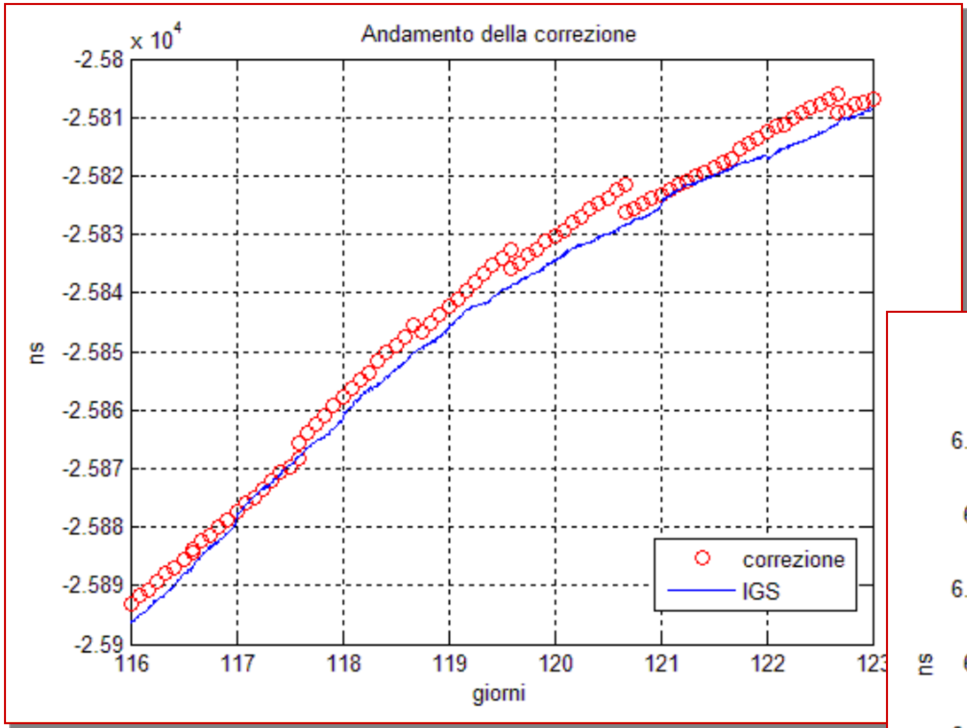
$$\delta t^{(S)} = a_{f0} + a_{f1} \cdot (t - t_{oc}) + a_{f2} \cdot (t - t_{oc})^2 + \Delta t_r$$

  - t_{oc} reference time for the clock model (seconds)
  - t satellite clock time (seconds, from beginning of GPS week)
  - Δt_r relativistic correction term (to be computed by the receiver!)

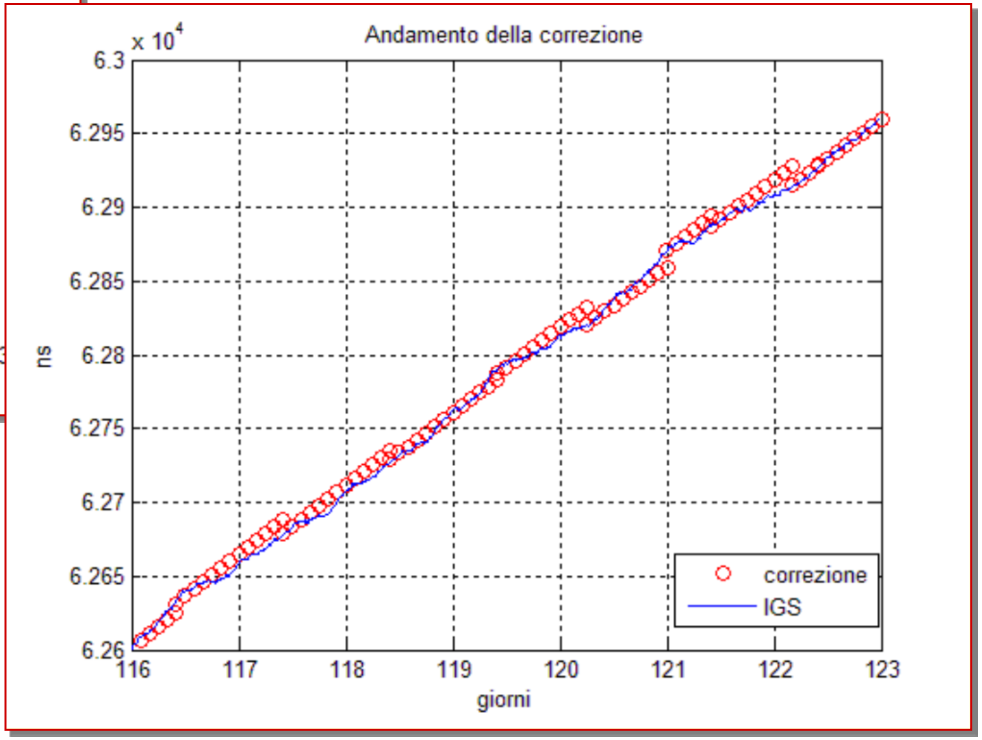
- The parameters of the model $\{a_{f0}, a_{f1}, a_{f2}, t_{oc}\}$ are uploaded to the satellite which broadcasts them in the Subframe 1 of the navigation message
- RMS error estimation of $\delta t^{(S)} \approx 5$ ns

How to get GPS Time? (cont'd)

satellite clock (cont'd)

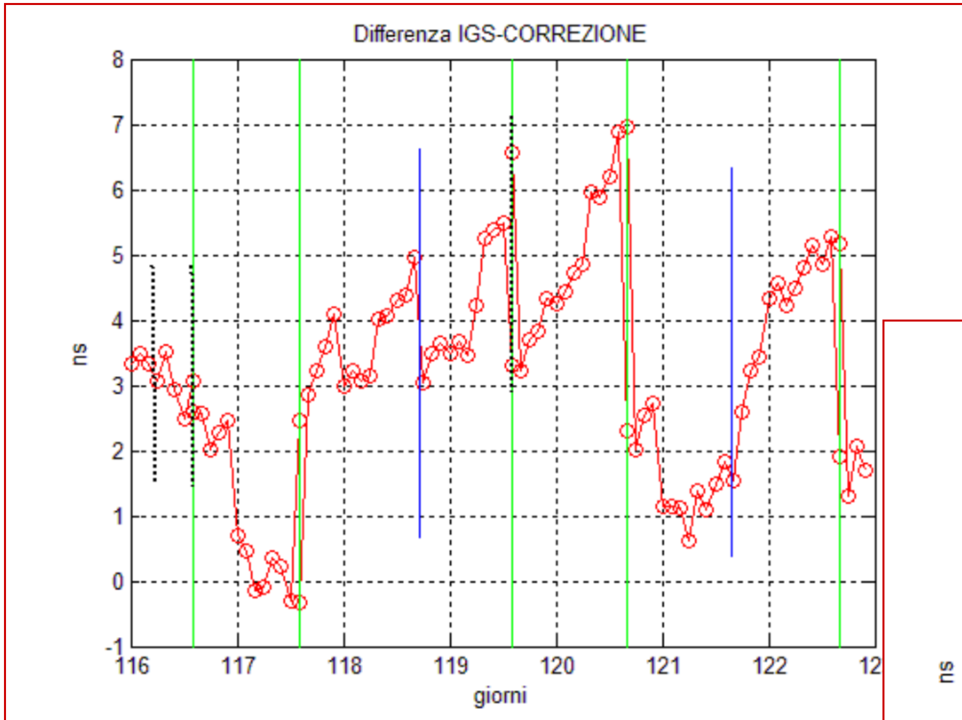


Satellite PRN 2 (Block IIR, Rb on-board clock)

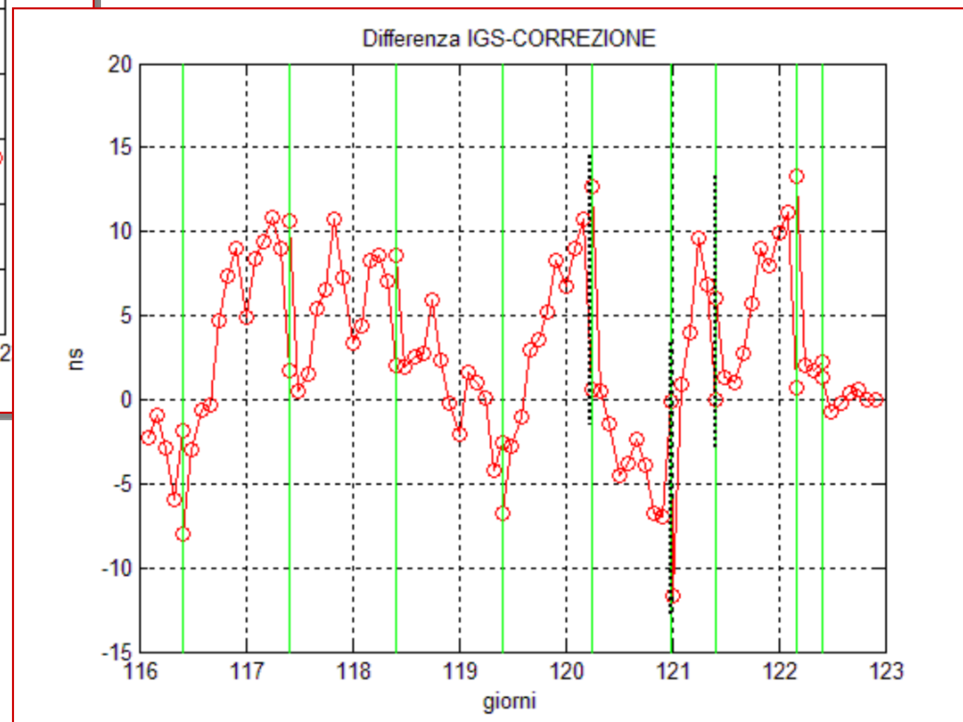


Satellite PRN 10 (Block IIA, Cs on-board clock)

satellite clock (cont'd)



Satellite PRN 2 (Block IIR, Rb on-board clock)



Satellite PRN 10 (Block IIA, Cs on-board clock)

How to get GPS Time? (cont'd)

ionosphere

$$\rho_R^{(S)}(t) = R_R^{(S)}(t,t-\tau) + c \cdot [\delta t_R(t) - \delta t^{(S)}(t-\tau)] + I_R^{(S)}(t) + T_R^{(S)}(t) + \varepsilon_R^{(S)}(t)$$

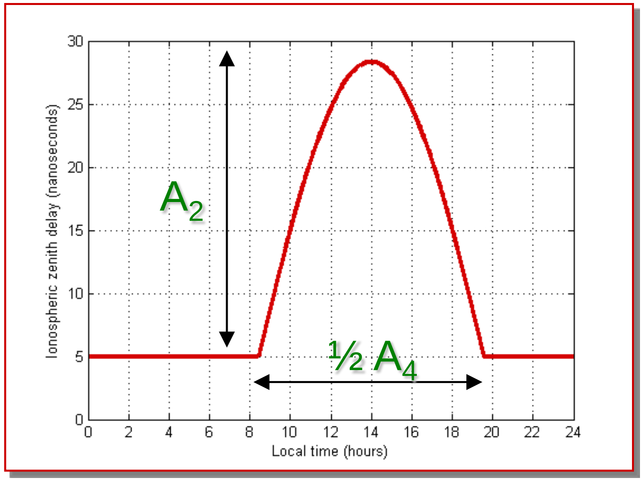
- Ionosphere major facts:
  - 50 km to 1000 km above the Earth
  - Dispersive medium (refraction index is dependent on the frequency)
    - Group velocity (v_g) differs from phase velocity (v_p)
    - Codes (i.e., C/A, P) experiences a group delay
    - Carrier (L1, L2) experience a phase advance
  - Ionosphere delay is dependent on the Total Electron Content (TEC)

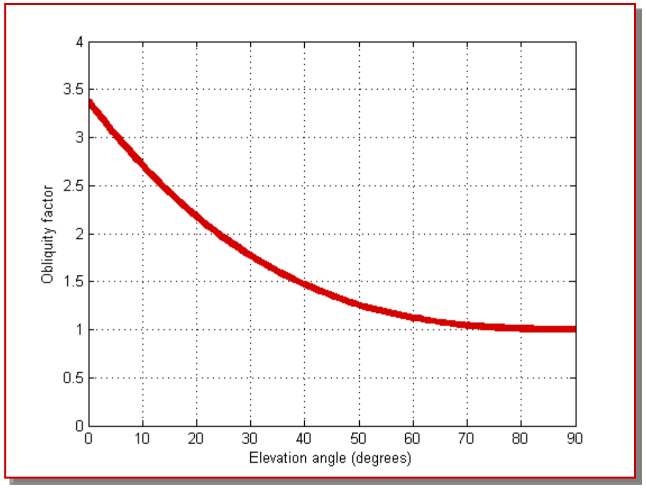
$$I_R^{(S)} \approx [40.3/(c \cdot f^2)] \cdot TEC$$

    - Delays in pseudorange measurement (code) and carrier phase measurement are equal in magnitude but opposite in sign $\Rightarrow$ "code-to-carrier divergence"

ionosphere (cont'd)

- Single-frequency receivers (L1 only or L2 only) are allowed to compute the ionospheric delay $I_R^{(S)}$
  - by using the Klobuchar model
    - empirical model representing the zenith delay as a constant value at nighttime and a half-cosine function in daytime
    - parameters are estimated by GPS CS and broadcast by each satellite in the page 18 of Subframe 4 of the navigation message
  - by combining the model with an obliquity factor (to be computed by the receiver!)
    - accounts for longer signal path length through the ionosphere wrt zenith path
    - depends on the satellite elevation angle

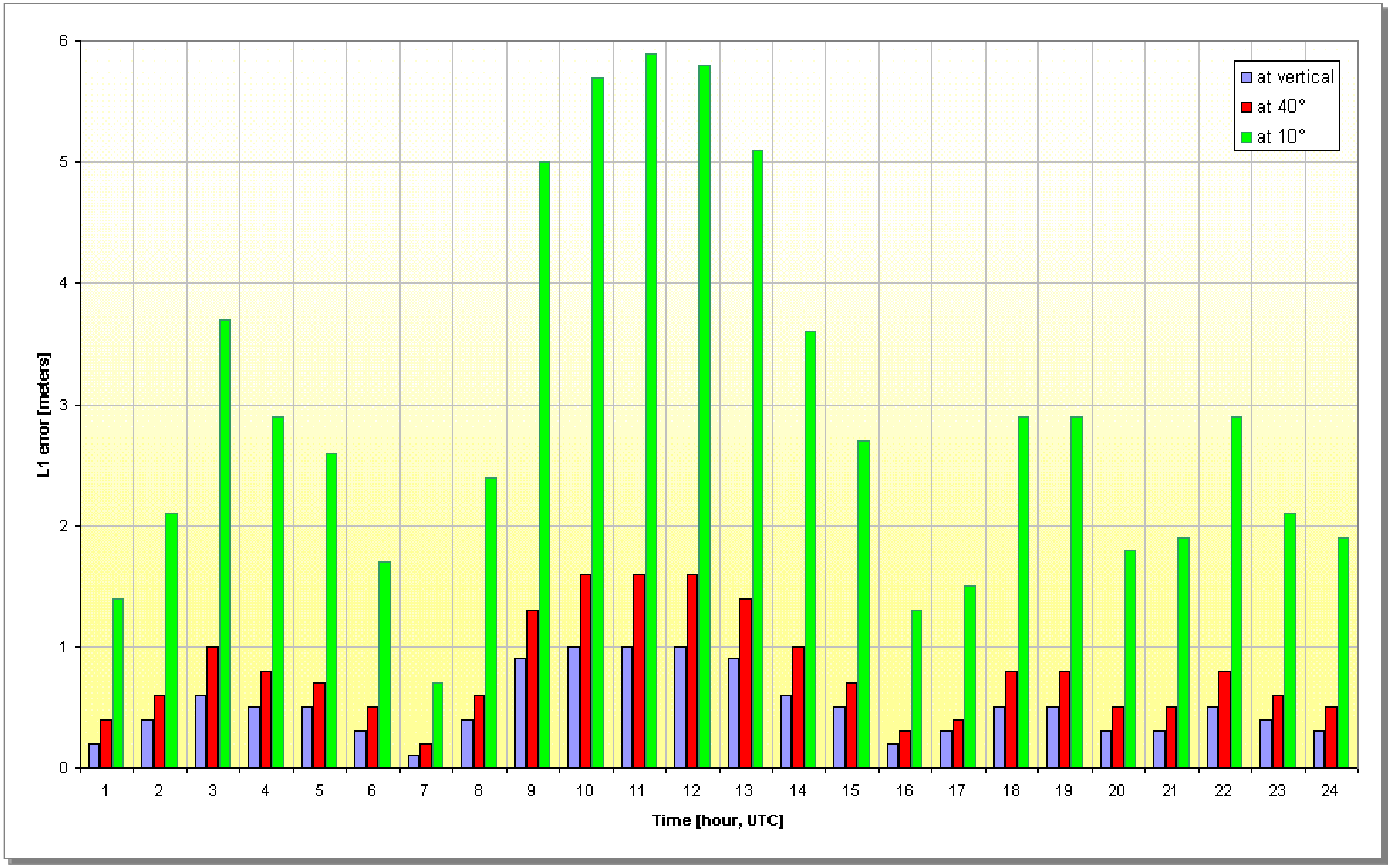




ionosphere (cont'd)

Comparison between the Klobuchar predicted TEC and the measured TEC at Brussels on an hourly basis.

Source: http://gpsatm.oma.be



How to get GPS Time? (cont'd)

ionosphere (cont'd)

- Dual-frequency receivers (L1 and L2) can compute the group delay and the phase advance directly from measurements on both frequencies
  - by building the ionosphere-free pseudorange measurement, which is actually a linear combination of dual-frequency measurements:

$$\rho_R{}^{(S)}{}_{free} = 2.546 \cdot \rho_R{}^{(S)}{}_{L1} - 1.546 \cdot \rho_R{}^{(S)}{}_{L2}$$

- The ionosphere-free pseudorange measurement is ideally not affected by ionospheric effect, nevertheless:
  - both modelling errors and unmodelled errors affecting dual-frequency pseudorange measurement fully result in $\rho_R{}^{(S)}{}_{free}$
  - $\rho_R{}^{(S)}{}_{free}$ is significantly noisier than each single-frequency measurement

$$(2.546^2 + 1.546^2)^{½} \approx 3$$

assuming that both dual-frequency measurements, $\rho_R{}^{(S)}{}_{L1}$ and $\rho_R{}^{(S)}{}_{L2}$, have the same noise

How to get GPS Time? (cont'd)

troposphere

$$\rho_R^{(S)}(t) = R_R^{(S)}(t,t-\tau) + c \cdot [\delta t_R(t) - \delta t^{(S)}(t-\tau)] + I_R^{(S)}(t) + T_R^{(S)}(t) + \varepsilon_R^{(S)}(t)$$

- Troposphere major facts:
  - 0 km to 50 km above the Earth
  - Refraction index is not dependent on the frequency (within L band)
    - Not a dispersive medium (n ≈ 1.0003 at sea level, approaching unity at upper end)
    - Both codes (i.e., C/A, P) and carrier (L1, L2) experience a delay
  - The tropospheric delay cannot be estimated from GPS measurements!
    - no parameters are provided by GPS CS in navigation message
    - resort to models to compensate for it
      - Saastamonien model
      - Hopfield model
    - implemented by the receiver firmware

How to get GPS Time? (cont'd)

QUESTION

- How can we compare versus GPS Time the reference clock of a stand-alone GPS (timing) receiver using the signals broadcasted by a single GPS satellite?

ANSWER

- Three steps procedure:
  - Perform pseudorange measurement
  - Correct (invert) pseudorange measurement
  - Calibrate Time measurement

"One-Way" measurement

How to get GPS Time? (cont'd)

$$\rho_R^{(S)} - \{R_R^{(S)} - c \cdot \delta t^{(S)} + I_R^{(S)} + T_R^{(S)}\} = c \cdot \delta t_R + \varepsilon'^{(S)}_R$$

$\rho_{corr,\ R}^{(S)}$

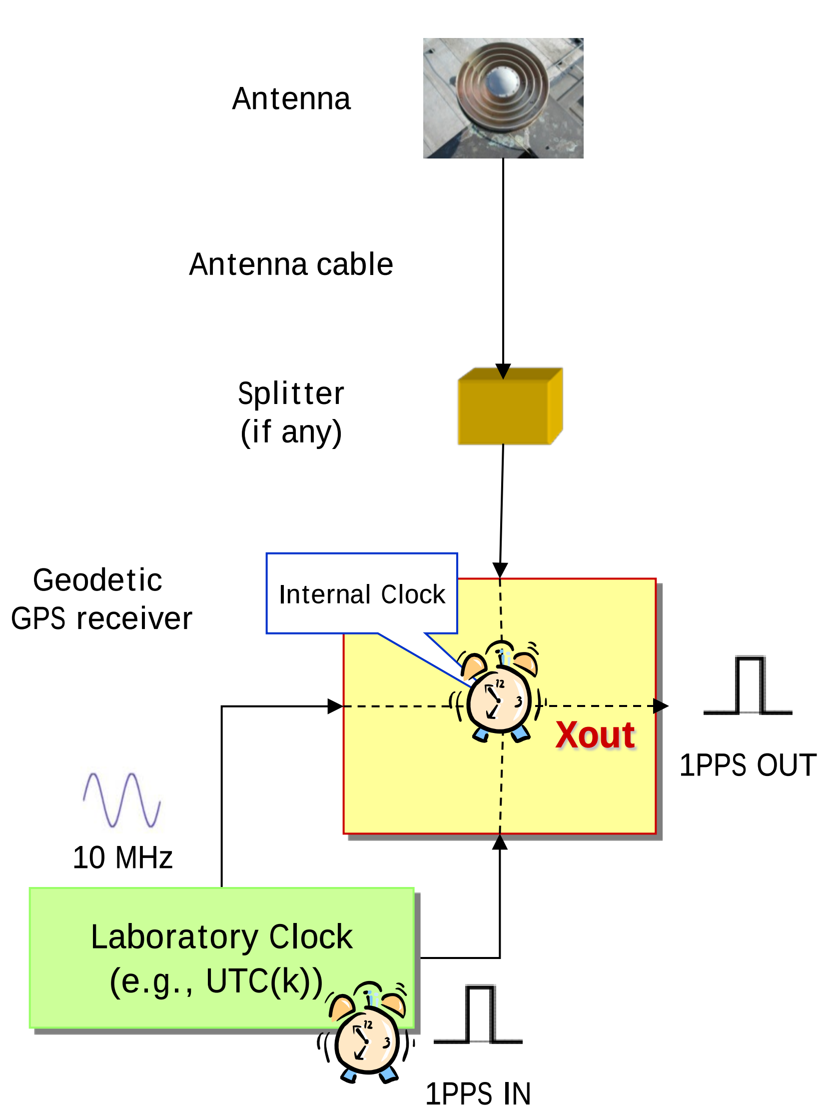
accounting for satellite clock offset and compensating for any known errors using parameters from navigation message

$$\varepsilon'^{(S)}_R = \varepsilon_R^{(S)} + \delta\varepsilon_{corr,R}^{(S)}$$

accounting for the combined effect of the residual errors after pseudorange correction

- From each "corrected" pseudorange $\rho_{corr,\ R}^{(S)}$, a noisy and biased estimate of the receiver clock offset versus GPS Time, δt_R, can be then achieved:
    - Noisy (precision)= noise on the pseudorange, "quality" of the correction, residuals, number of satellites (more pseudoranges);
    - Biased (accuracy)= Hardware Delays (calibration).

How to get GPS Time? (cont'd)



X_S Antenna delay
(for both GPS frequencies → X_{S1}, X_{S2})

X_C Antenna cable delay

X_D Splitter delay
(including any further short cable delay)

X_R Receiver internal delay
from the "internal reference" of receiver
(for both GPS frequencies → X_{R1}, X_{R2})

X_O Offset from "internal reference"
to 1PPS input

X_P Offset from 1PPS input to laboratory reference
(signals distribution chain)

How to get GPS Time? (cont'd)

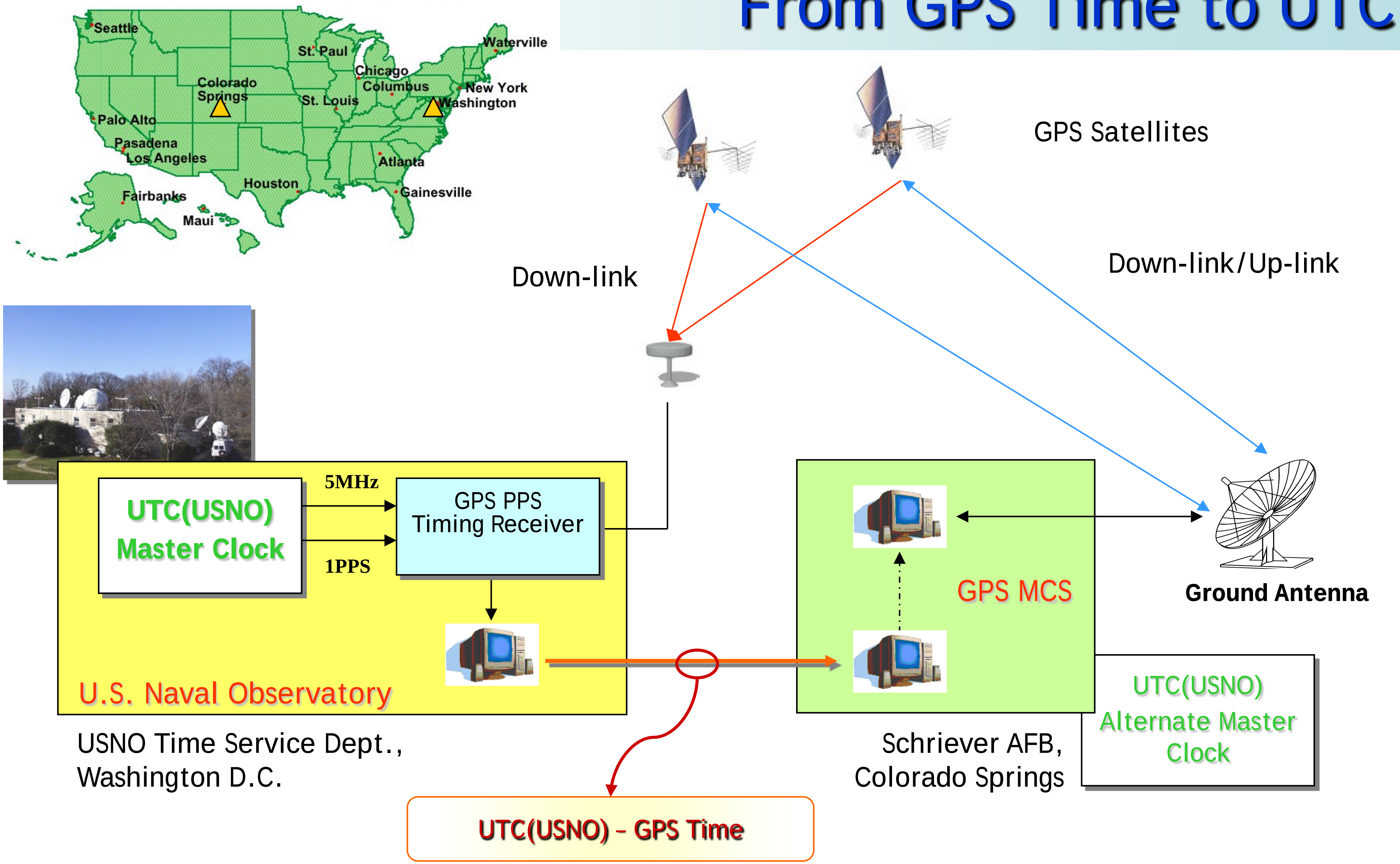
- In order to get the actual receiver clock offset wrt GPS Time $\delta t_{R,bias}$ in a "one-way" measurement, the hardware delay $\delta t_{R,hw}$ (due to the antenna and its preamplifier, the antenna cable and the receiver hardware) has to be estimated and then removed

  $\Rightarrow$ Calibration

$$\delta t_R = \delta t_{R,hw} + \delta t_{R,bias}$$

- Two classes of calibration methods
  - Absolute calibration (U=1 ns)
    - End-to-end biases estimation (preferred, but complex)
  - Differential calibration (U=2-3 ns)
    - Side-by-side comparison with an absolutely calibrated device (easier than absolute)

From GPS Time to UTC



Source: M. Miranian, "*USNO Report*", CGSIC Timing Subcommittee, 2001

From GPS Time to UTC (cont'd)

δt_{UTC} offset between UTC(USNO) and GPS Time: $\delta t_{UTC} = t_{GPS} - t_{UTC(USNO)}$ (in seconds)

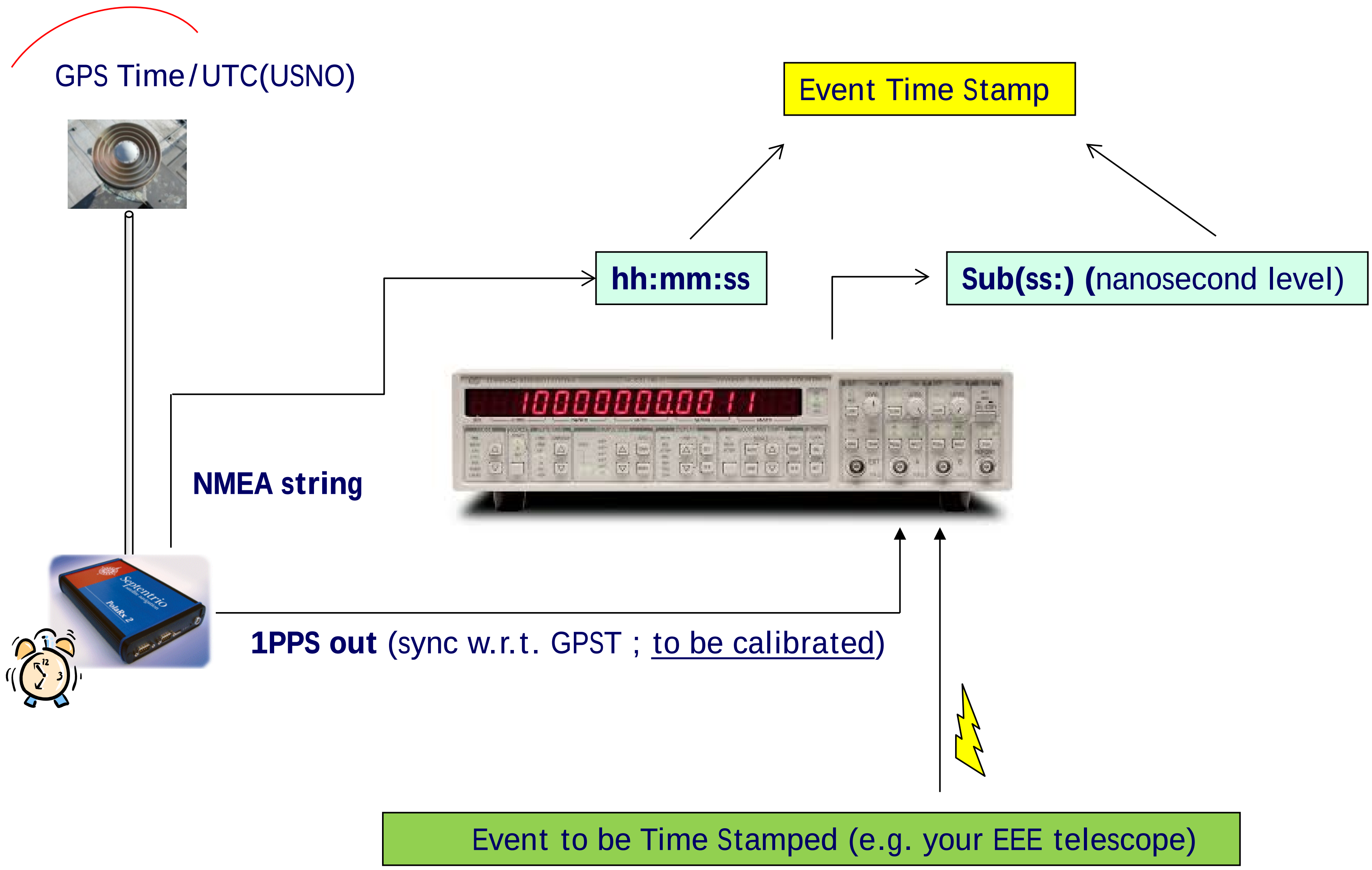
- Predicted by the GPS Control Segment (thanks to data from USNO)
- Modeled as a linear function over a time interval

$$\delta t_{UTC} = A_0 + A_1 \cdot (t_{GPS} - t_{ot}) + \Delta t_{LS}$$

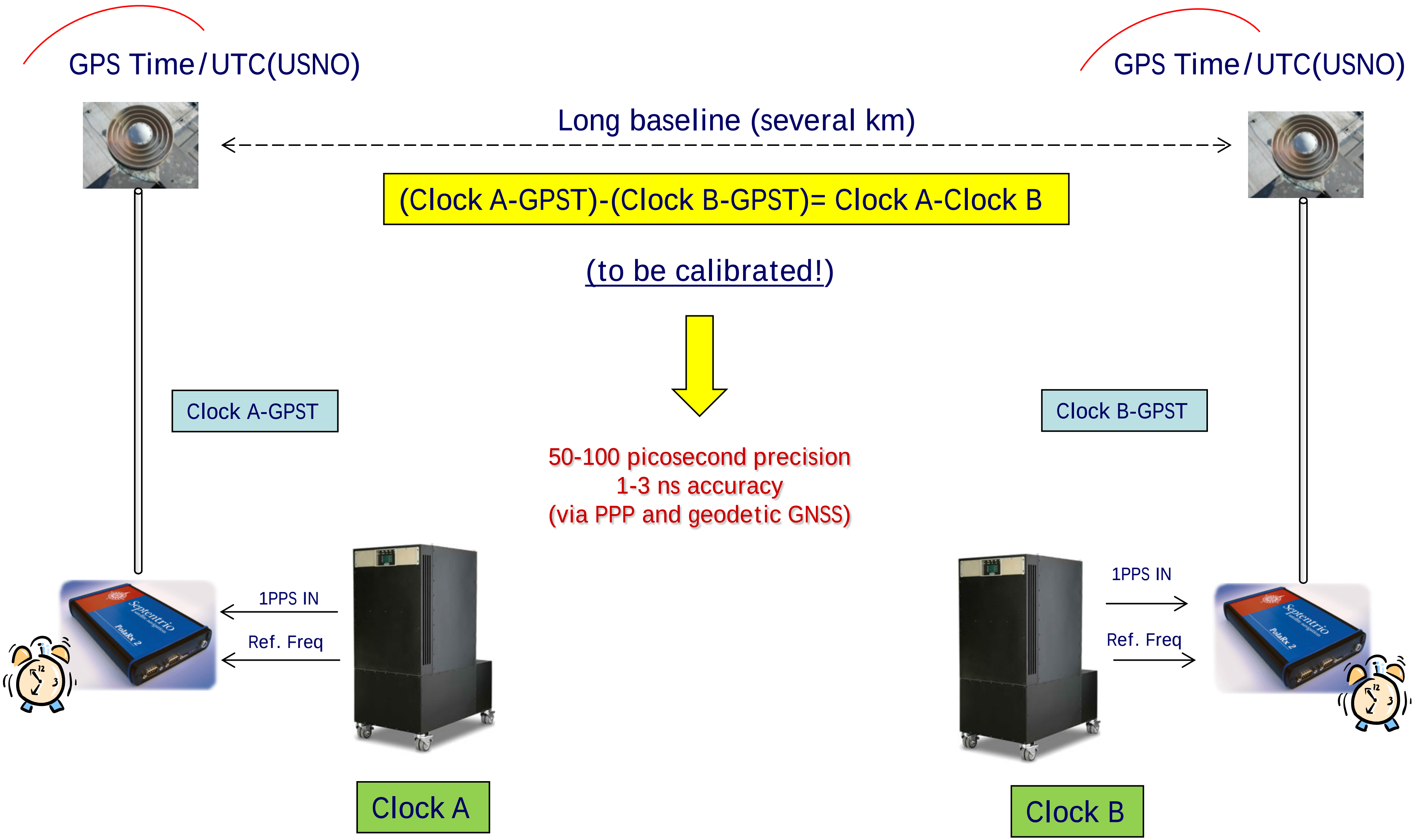
  - t_{ot} reference time for UTC data (seconds, from beginning of GPS week WN_t)
  - t_{GPS} GPS Time as estimated by the user (seconds, from beginning of GPS week)
  - Δt_{LS} delta time due to leap seconds (added to UTC since 1980; now Δt_{LS} = 18 seconds)

- The parameters of the model $\{A_0, A_1, t_{ot}, \Delta t_{LS}, WN_t\}$ are uploaded to the satellite which broadcasts them in the page 18 of Subframe 4 of the navigation message
- RMS error estimation of $\delta t_{UTC} \approx 5 \div 10$ ns

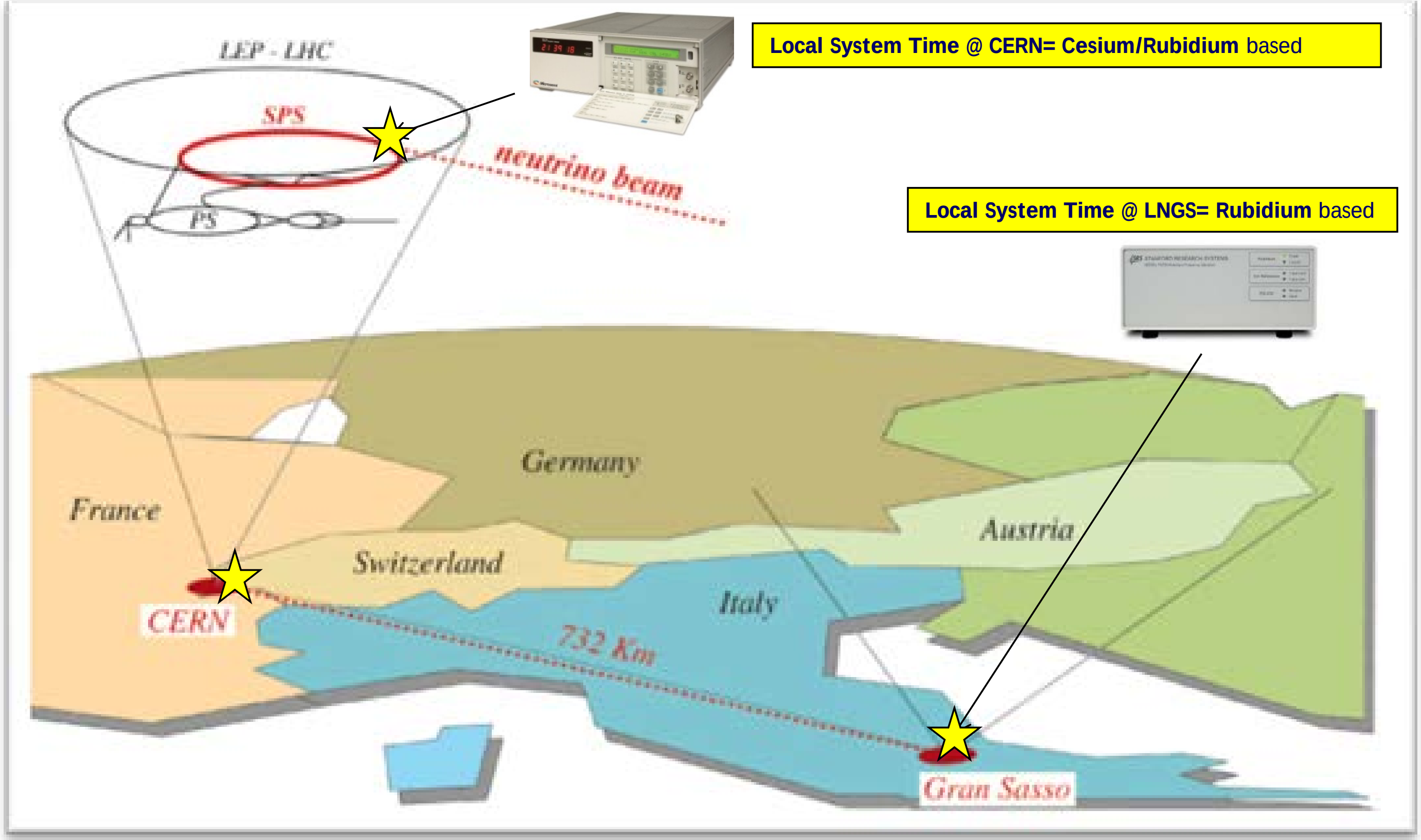
GNSS for timing applications (1/2)



GNSS for timing applications (2/2)

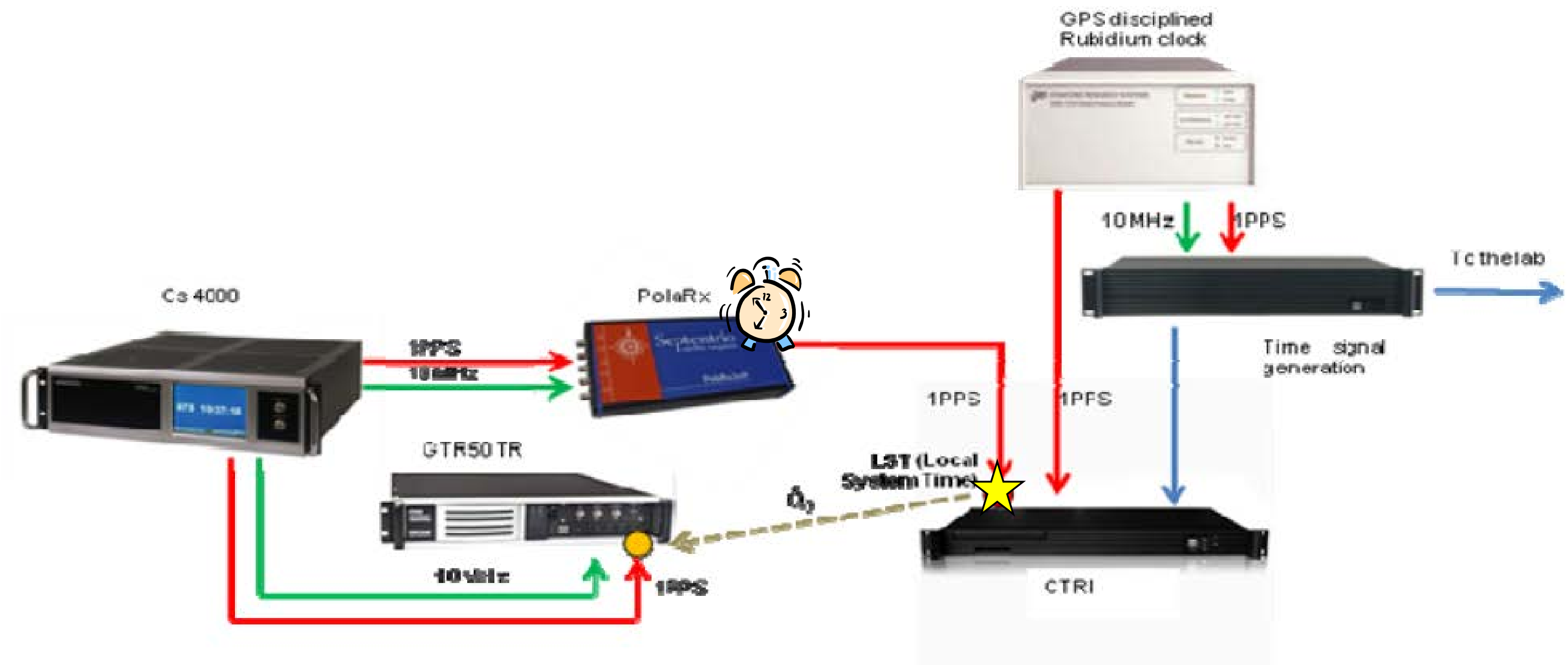


Timing in the neutrinos speed measurement (1/5)



- LST@CERN and LST@LNGS not synchronized (link not calibrated)
- Timing systems at CERN and at LNGS to be <u>calibrated</u>

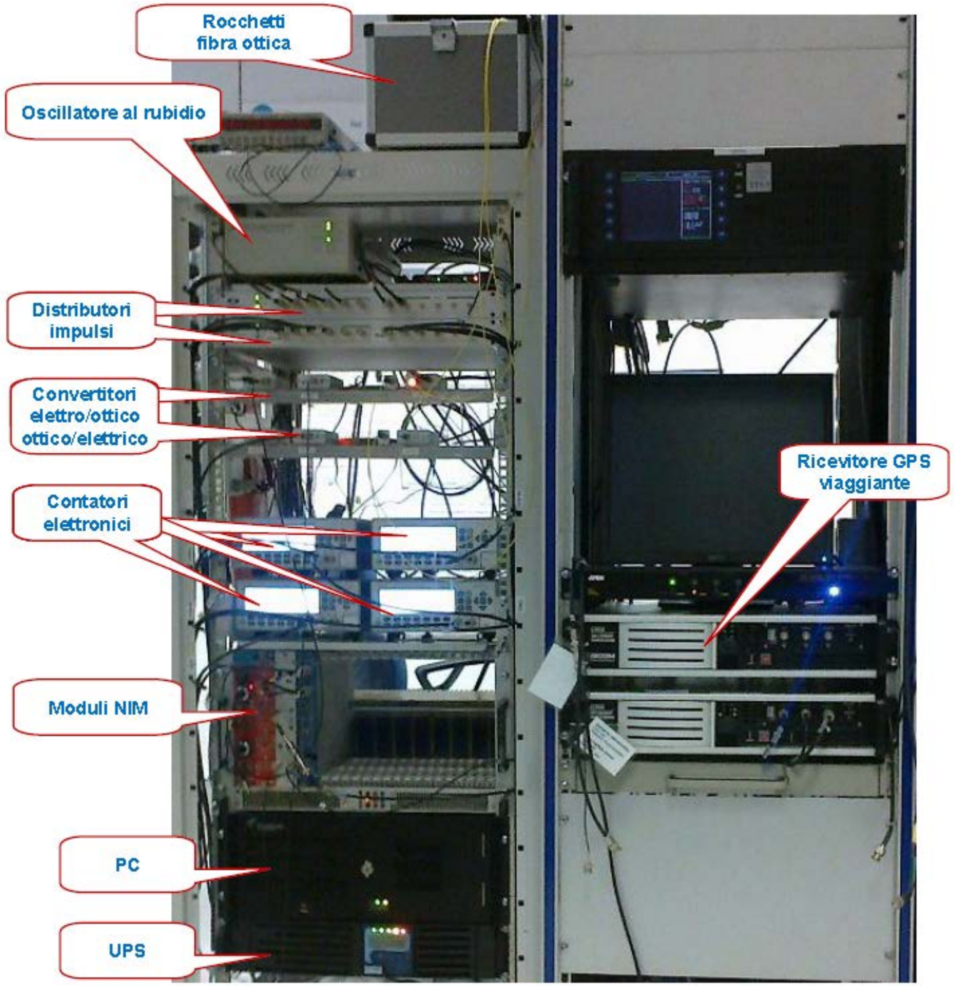
Timing in the neutrinos speed measurement (2/5)

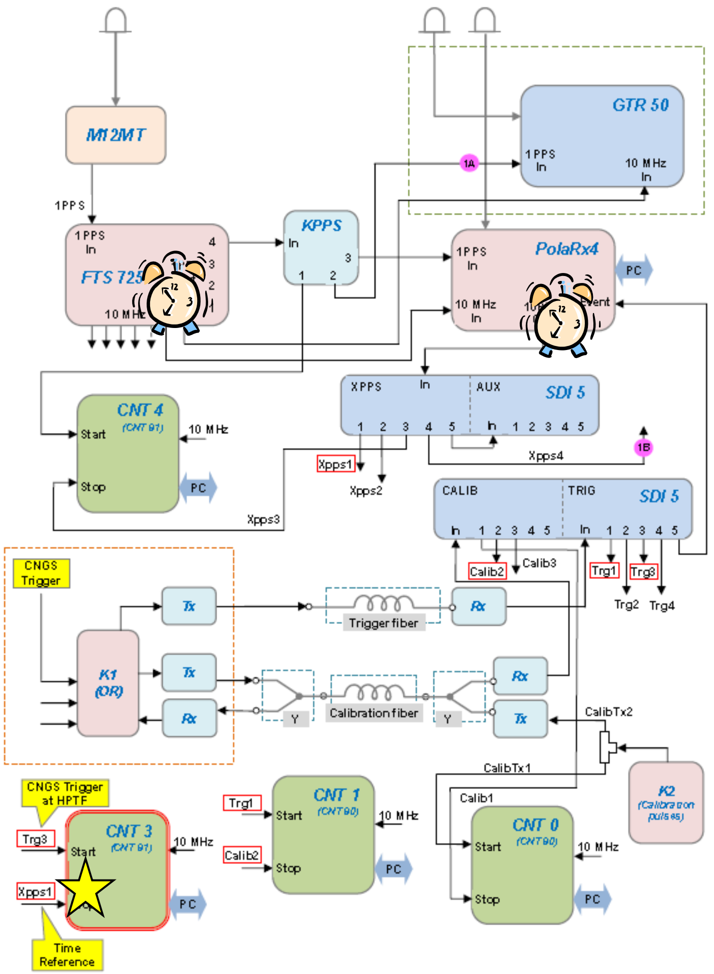


$$\Delta_{CERN}=(LST_{CERN}-GPST)_{TR}-(Rxclock_{CERN}-GPST)$$

V. Pettiti, G. Cerretto, «Taratura e controllo di apparecchiature per la datazione di eventi», Relazione INRIM (Certificato di Taratura) N. 12-0391-01 emessa il 15/06/2012, Committente INFN-LNGS (Laboratori Nazionali del Gran Sasso)

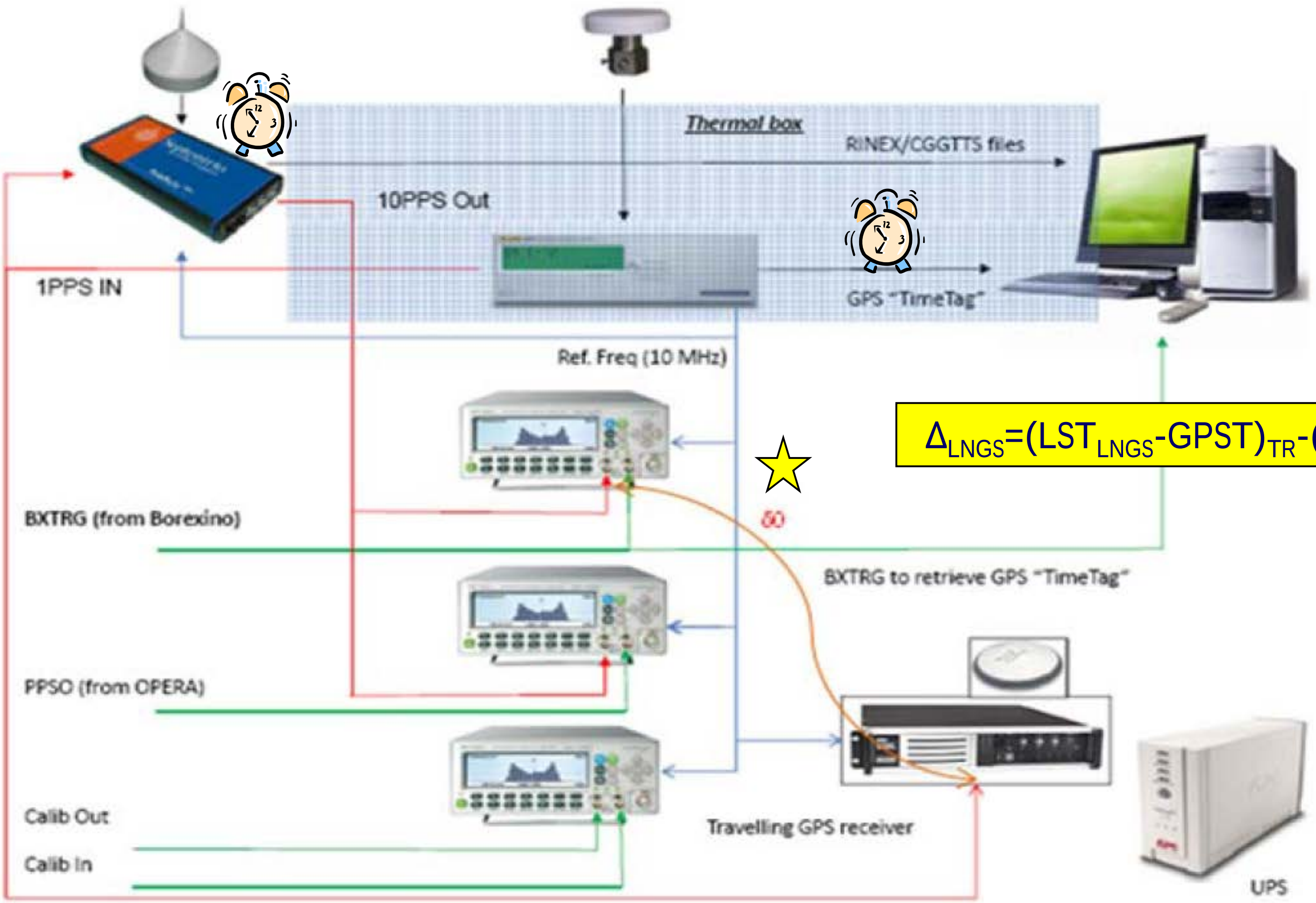
Timing in the neutrinos speed measurement (3/5)





V. Pettiti, G. Cerretto, «Taratura e controllo di apparecchiature per la datazione di eventi», Relazione INRIM (Certificato di Taratura) N. 12-0391-01 emessa il 15/06/2012, Committente INFN-LNGS (Laboratori Nazionali del Gran Sasso)

Timing in the neutrinos speed measurement (4/5)



$\Delta_{LNGS}=(LST_{LNGS}\text{-}GPST)_{TR}\text{-}(Rxclock_{LNGS}\text{-}GPST)$

V. Pettiti, G. Cerretto, «Taratura e controllo di apparecchiature per la datazione di eventi», Relazione INRIM (Certificato di Taratura) N. 12-0391-01 emessa il 15/06/2012, Committente INFN-LNGS (Laboratori Nazionali del Gran Sasso)

Timing in the neutrinos speed measurement (5/5)

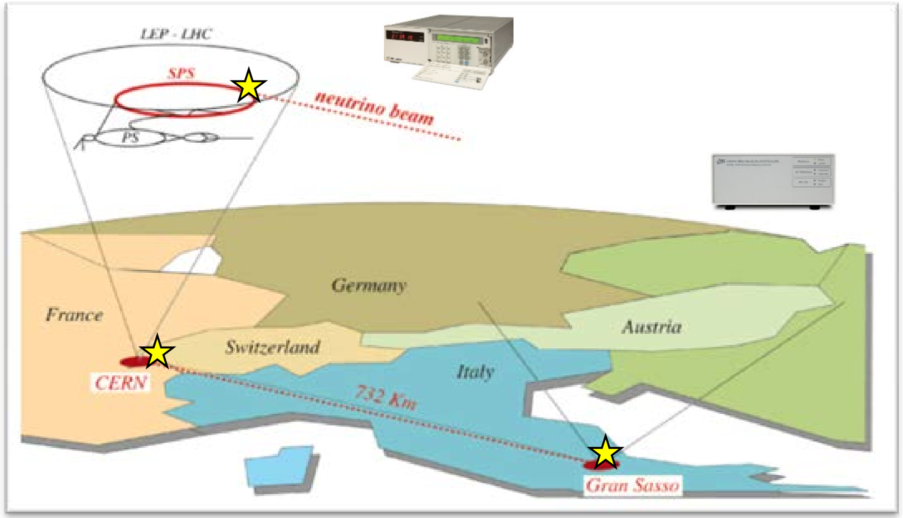
$\Delta_{CERN}=(LST_{CERN}\text{-}GPST)_{TR}-(Rxclock_{CERN}\text{-}GPST)$

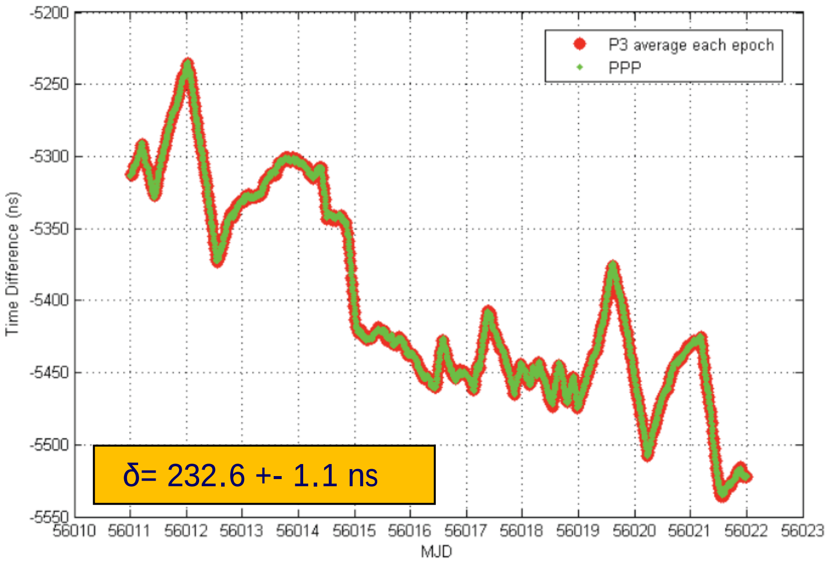
$\Delta_{LNGS}=(LST_{LNGS}\text{-}GPST)_{TR}-(Rxclock_{LNGS}\text{-}GPST)$

$\delta=\Delta_{LNGS}-\Delta_{CERN}$

$\delta=(LST_{LNGS}\text{-}GPST)_{TR}-(Rxclock_{LNGS}\text{-}GPST)-(LST_{CERN}\text{-}GPST)_{TR}+(Rxclock_{CERN}\text{-}GPST)$

$LST_{CERN}-LST_{LNGS}=(Rxclock_{CERN}\text{-}GPST)-(Rxclock_{LNGS}\text{-}GPST)-\delta$





B. Caccianiga et al, «GPS-based CERN-LNGS time link for Borexino», Journal of Instrumentation, Volume 7, August 2012